Optimal Vehicle Stability Control with Driver Input and Bounded Uncertainties

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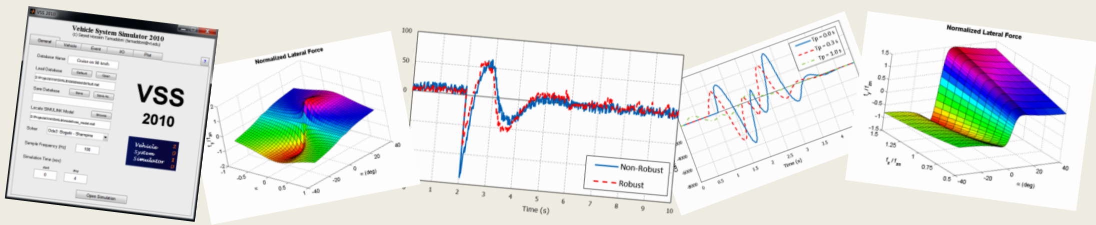


contents

- Objectives
- Dynamics Modeling

  - Vehicle/Tire Dynamics

  - Vehicle Control Systems

  - Driver Directional Control Model
- Vehicle Control Design Based on Game Theory

  - Continuous-Time Interaction Model

  - Discrete-Time Interaction Model

  - Sensitivity Analysis

  - Robust Interaction Model
- Conclusions

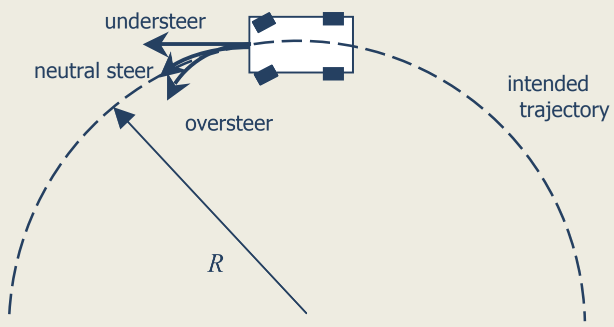
research objectives

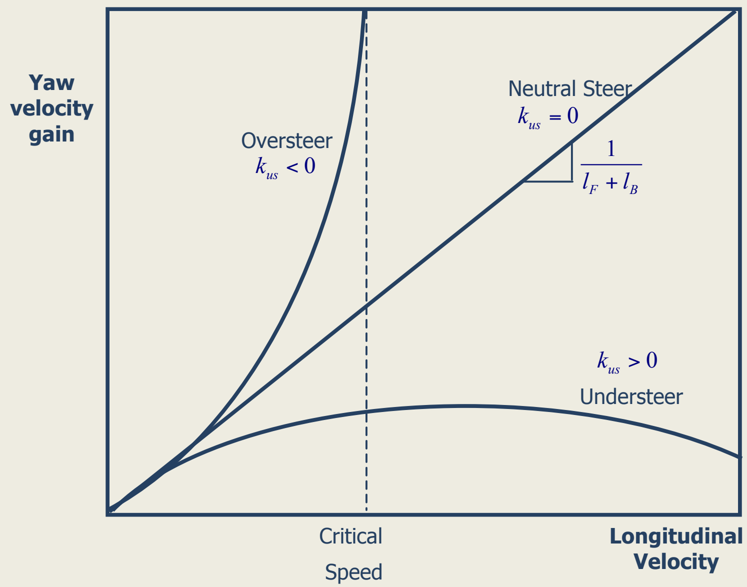
- Develop a novel vehicle stability controller that encompasses the driver dynamics
- Develop a vehicle driver-controller interaction model
- Evaluate the effectiveness of the suggested vehicle driver-controller model *
- Evaluate the robustness of the proposed interaction model, and design a robust controller

* It needs a validated vehicle model that captures the essential dynamics

handling concept

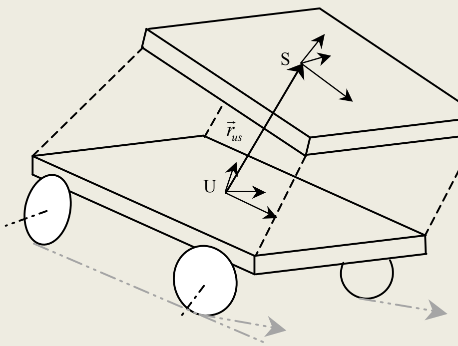
- Cornering: ability of vehicle to travel on curved path
- Yaw velocity gain: $\dfrac{\dot{\theta}_z}{\delta_F} = \dfrac{v_x}{(l_F + l_B)\left(1 + k_{us} v_x^2\right)}$
- Understeering coefficient: $k_{us} = \dfrac{m\left(l_B c_{\alpha B} - l_F c_{\alpha F}\right)}{\left(l_F + l_B\right)^2 c_{\alpha F} c_{\alpha B}}$

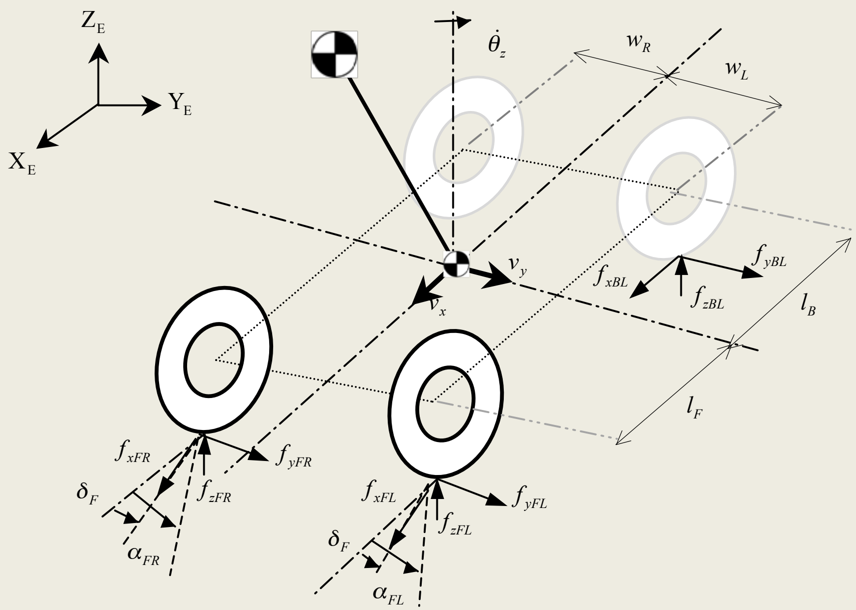




vehicle dynamics

- **Body dynamics**
- Wheel/Tire dynamics





Longitudinal: $m\left(\dot v_x - v_y\dot\theta_z\right) - m_s h_s\left(\theta_x\ddot\theta_z + 2\dot\theta_x\dot\theta_z\right) = \sum f_x\cos\delta - \sum f_y\sin\delta$

Lateral: $m\left(\dot v_y + v_x\dot\theta_z\right) - m_s h_s\left(\theta_x\dot\theta_z^2 - \ddot\theta_x\right) = \sum f_x\sin\delta + \sum f_y\cos\delta$

Roll: $m_s h_s\left(\dot v_y + v_x\dot\theta_z\right) + \left(i_{sx} + m_s h_s^2\right)\ddot\theta_x + \left(i_{sz} - i_{sy} - m_s h_s^2\right)\theta_x\dot\theta_z^2 = -k_x\theta_x - c_x\dot\theta_x + m_s h_s g\sin\theta_x$

Yaw: $-m_s h_s\theta_x\left(\dot v_x - v_y\dot\theta_z\right) + \left(i_{uz} + i_{sz}\right)\ddot\theta_z = \sum l' f_x\sin\delta - \sum l' f_y\cos\delta + \sum w' f_x\cos\delta - \sum w' f_y\sin\delta$

Multibody Equations of Motion:

$$\begin{cases} (m_u + m_s)\,{}^U\vec a_u + m_s\,{}^S\vec a_s = \sum_i {}^U\vec f_i \\ {}^U\dot H_u + {}^S\dot H_s + {}^U\vec r_{us}\times\left(m_s\,{}^S\vec a_s\right) = \sum_i {}^U\vec\tau_i \end{cases}$$

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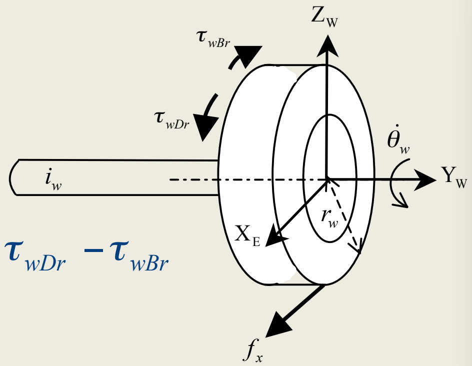
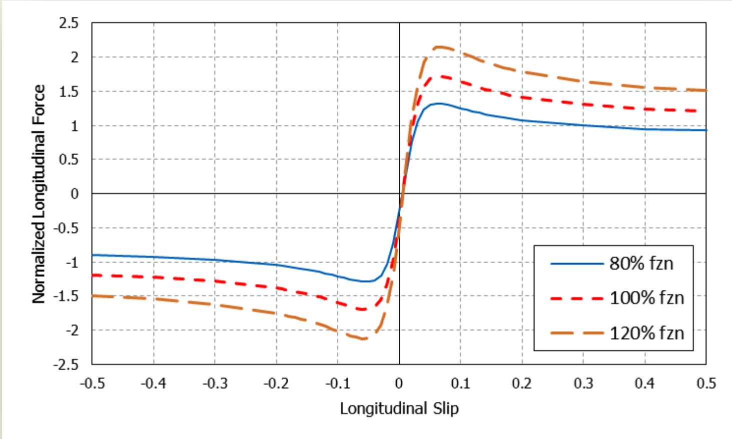
tire dynamics

- Body dynamics
- **Wheel/Tire dynamics**

$$i_w\ddot{\theta}_w = -f_x r_w + \tau_{wDr} - \tau_{wBr}$$





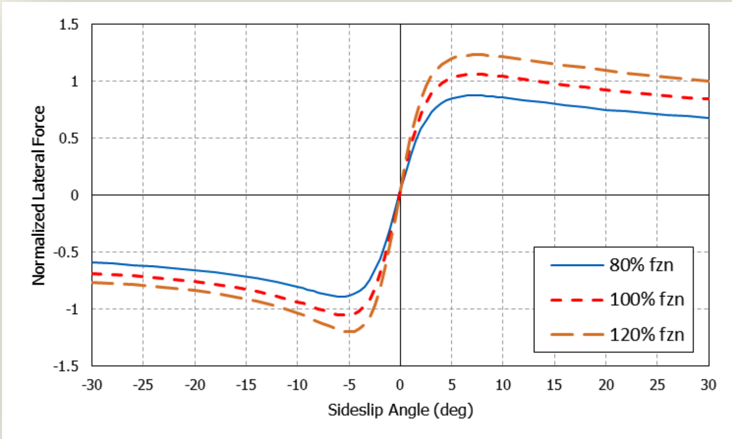
❖ *Pacejka "Magic Formula" tire model*

$$f = f_0(x, f_z) = D.\sin\left[C.\arctan\left(B.x' - E.\left(B.x' - \arctan\left(B.x'\right)\right)\right)\right] + S_V$$

❖ *LuGre tire model*

$$f_x = \left(\sigma_0\mu_n + \sigma_1\dot{\mu}_n + \sigma_2 v_r\right) f_z$$

$$\dot{\mu}_n + \left(\frac{\sigma_0\left|v_r\right|}{\mu_c + \left(\mu_s - \mu_c\right)e^{-\left|v_r/v_s\right|^{\alpha}}} + \bar{\kappa}\left|r_w\dot{\theta}_w\right|\right)\mu_n = v_r$$



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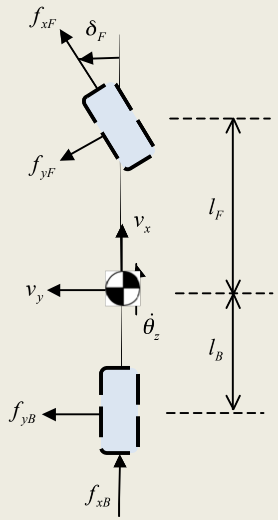
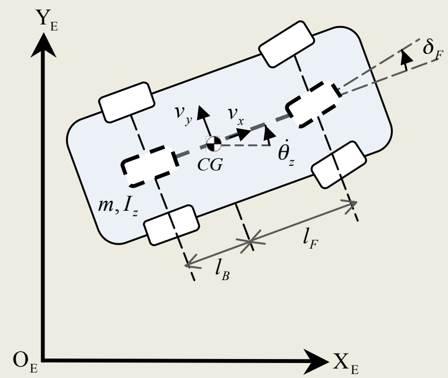
linear 2-DOF vehicle model

- lateral position, <u>lateral velocity</u>, yaw angle, <u>yaw rate</u>

$$x = \left(y \quad \dot{y} \quad \theta_z \quad \dot{\theta}_z \right)^T$$





$$\dot{x} = \begin{bmatrix} 0 & 1 & v_x & 0 \\ 0 & -\dfrac{c_{\alpha F} + c_{\alpha B}}{m v_x} & 0 & -v_x - \dfrac{l_F c_{\alpha F} - l_B c_{\alpha B}}{m v_x} \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{l_F c_{\alpha F} - l_B c_{\alpha B}}{i_z v_x} & 0 & -\dfrac{l_F^2 c_{\alpha F} + l_B^2 c_{\alpha B}}{i_z v_x} \end{bmatrix} x + \begin{bmatrix} 0 \\ \dfrac{c_{\alpha F}}{r_{st} m} \\ 0 \\ \dfrac{l_F c_{\alpha F}}{r_{st} i_z} \end{bmatrix} \delta_{sw}$$

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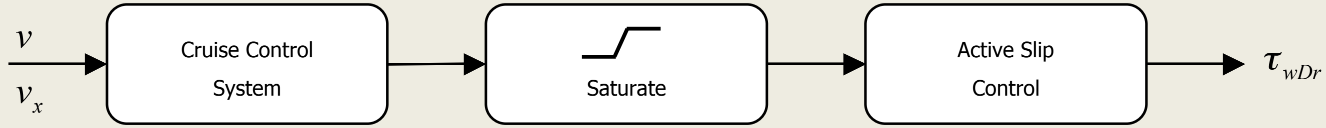
traction/braking control systems

- Cruise Control
- Differential Braking Control



$$\tau_{wDr} = \begin{cases} \frac{1}{2} \mathcal{L}_{PID}\left(v_{x,des} - v_x\right) . \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix}^T & , v_{xd} > v_x \\ 0_{1\times 4} & , v_{xd} \leq v_x \end{cases}$$

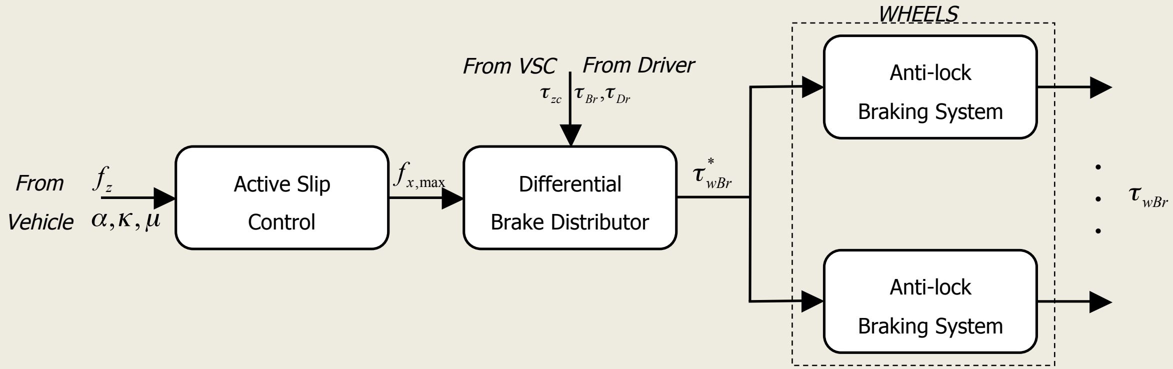
$$\mathcal{L}_{PID} = k_{pcc} + k_{icc}\int_0^t dt + k_{dcc}\frac{d}{dt}$$

$$ASC = \begin{cases} 0 & , v_x \leq 0.7 v_{avg} \\ \frac{1}{0.15}\left(\frac{v_x}{v_{avg}} - 0.7\right) & , 0.7 v_{avg} < v_x < 0.85 v_{avg} \\ 1 & , v_x \geq 0.85 v_{avg} \end{cases}$$

$$v_{avg} = \left(\frac{r_w \dot{\theta}_{wBR} + r_w \dot{\theta}_{wBL}}{2}\right)$$

traction/braking control systems

- Cruise Control
- Differential Braking Control



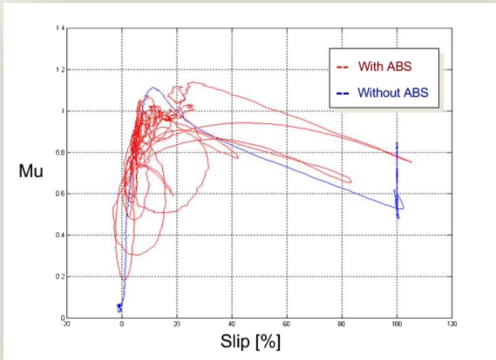
$$f_{x,\max} = \begin{cases} 0 & , \ \kappa \geq 0.8 \\ \mu f_z \left(\dfrac{\kappa}{\sqrt{\kappa^2 + \tan^2(\alpha)}} \right) & , \ \kappa < 0.8 \end{cases}$$

$$\tau^*_{wBr,i} = \max\left(r_w f_{x,i} + \tau_{wDr,i}, \tau_{wBr,\max} \right)$$

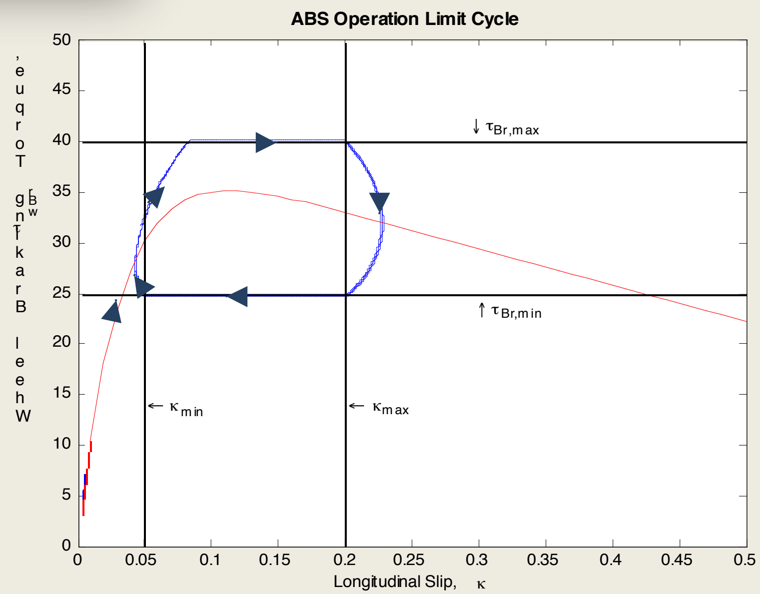
$$\begin{cases} f_{xFR} + f_{xFL} + f_{xBR} + f_{xBL} = \dfrac{\tau_{accel} - \tau_{decel}}{r_w} \\ f_{xFR} - f_{xFL} + f_{xBR} - f_{xBL} = \dfrac{\tau_{zc}}{w} \\ \left| f_{xFR} \right| \leq f_{xFR,\max} \\ \left| f_{xFL} \right| \leq f_{xFL,\max} \\ \left| f_{xBR} \right| \leq f_{xBR,\max} \\ \left| f_{xBL} \right| \leq f_{xBL,\max} \end{cases}$$

anti-lock braking system

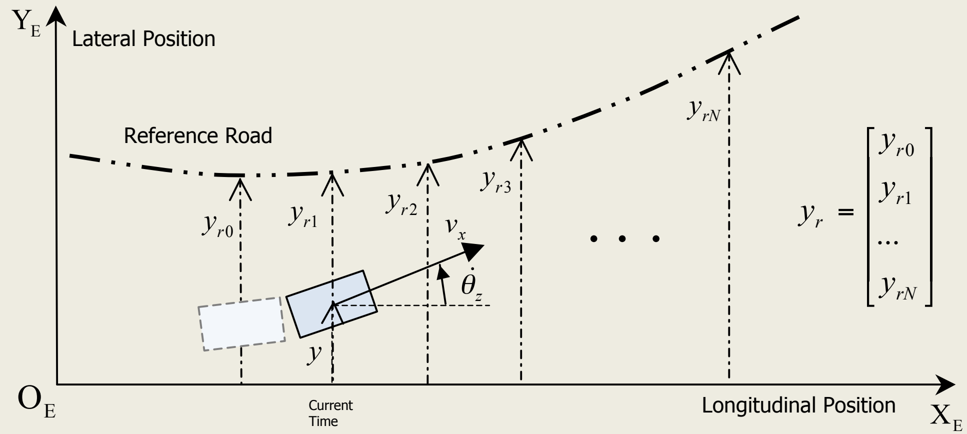
- prevents wheel lock-up during braking
- maintains vehicle stability and steering



$$
\begin{cases}
\tau_{wBr} = \tau_{wBr}^{*} & ,\kappa < \kappa_{\min} \\
\dot{\tau}_{wBr} = k_b & ,\kappa_{\min} < \kappa \le \kappa_{\max} \ \& \ \tau_{wBr,\min} \le \tau_{wBr} \le \tau_{Br,\max} \\
\dot{\tau}_{wBr} = 0 & ,\kappa_{\min} < \kappa \le \kappa_{\max} \ \& \ \tau_{wBr} \ge \tau_{Br,\max} \\
\dot{\tau}_{wBr} = 0 & ,\kappa_{\min} < \kappa \le \kappa_{\max} \ \& \ \tau_{wBr} \le \tau_{Br,\min} \\
\dot{\tau}_{wBr} = -k_b & ,\kappa > \kappa_{\max}
\end{cases}
\begin{matrix} \text{(A)} \\ \text{(B)} \\ \text{(C)} \\ \text{(D)} \\ \text{(E)} \end{matrix}
$$



driver steering control model



Linear Bicycle Model :

$$x_d^+ = \mathbf{A}_d x_d + \mathbf{B}_d \delta_{sw}$$

Road Preview Model :

$$y_r^+ = \underbrace{\begin{bmatrix} \mathbf{0}_{N\times 1} & \mathbf{I}_{N\times N} \\ 0 & \mathbf{0}_{1\times N} \end{bmatrix}}_{\mathbf{A}_r} y_r + \underbrace{\begin{bmatrix} \mathbf{0}_{N\times 1} \\ 1 \end{bmatrix}}_{\mathbf{B}_r} y_{r(N+1)}$$

$$\underbrace{\begin{bmatrix} x_d^+ \\ y_r^+ \end{bmatrix}}_{z^+} = \underbrace{\begin{bmatrix} \mathbf{A}_d & 0 \\ 0 & \mathbf{A}_r \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x_d \\ y_r \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} \mathbf{B}_d \\ 0 \end{bmatrix}}_{\mathbf{B}} \delta_{sw} + \underbrace{\begin{bmatrix} 0 \\ \mathbf{B}_r \end{bmatrix}}_{\mathbf{E}} y_{r(N+1)}$$

driver–controller interactions

- vehicle stability =

  driver decisions **+** controller performance

- Interaction modeling of driver-controller:

  - independent subsystems

  - collaborative subsystems

- In this research, the interaction modeling of **driver steering control** and **vehicle yaw control** is studied based on "<u>Linear Quadratic Game Theory</u>" approach.

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continuous-time interaction model

- Linear bicycle model:

$$\dot{x}(t) = \mathbf{A}_c x(t) + \mathbf{B}_{c1} \underbrace{\delta_{sw}(t)}_{u_1} + \mathbf{B}_{c2} \underbrace{M_{zc}(t)}_{u_2}$$

$$x(t) = \begin{bmatrix} y & v_y & \theta_z & \dot{\theta}_z \end{bmatrix}^T$$

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & V_x & 0 \\ 0 & -\dfrac{C_{\alpha F} + C_{\alpha B}}{MV_x} & 0 & -V_x - \dfrac{L_F C_{\alpha F} - L_B C_{\alpha B}}{MV_x} \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{L_F C_{\alpha F} - L_B C_{\alpha B}}{I_z V_x} & 0 & -\dfrac{L_F^2 C_{\alpha F} + L_R^2 C_{\alpha B}}{I_z V_x} \end{bmatrix}, \mathbf{B}_{c1} = \begin{bmatrix} 0 \\ \dfrac{C_{\alpha F}}{r_{st} M} \\ 0 \\ \dfrac{L_F C_{\alpha F}}{r_{st} I_z} \end{bmatrix}, \mathbf{B}_{c2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{I_z} \end{bmatrix}$$

- Objective function:

$$J_i(u_1, u_2) = \int_0^t \left(x^T \mathbf{Q}_i x + u_1^T \mathbf{R}_{i1} u_1 + u_2^T \mathbf{R}_{i2} u_2 \right) dt$$

- Nash equilibrium:

$$\begin{cases} J_1(u_1, u_2^*) \geq J_1(u_1^*, u_2^*) \\ J_2(u_1^*, u_2) \geq J_2(u_1^*, u_2^*) \end{cases}$$

LQ Game Theory

- Forming "Hamiltonian" as

$$\mathbf{H}_i\left(x,u_1,u_2,\mathbf{P}_i\right)=x^T\mathbf{Q}_i x+u_1^T\mathbf{R}_{i1}u_1+u_2^T\mathbf{R}_{i2}u_2+\mathbf{P}_i^T\left(\mathbf{A}_c x+\mathbf{B}_{c1}u_1+\mathbf{B}_{c2}u_2\right)$$

subject to

$$\frac{\partial}{\partial u_i}\mathbf{H}_i\left(x^*,u_1^*,u_2^*,\mathbf{P}_i\right)=0 \quad , \quad \frac{d}{dt}\mathbf{P}_i=-\frac{\partial}{\partial x}\mathbf{H}_i\left(x^*,u_1^*,u_2^*,\mathbf{P}_i\right)$$

where

$$\dot{x}^*(t)=\mathbf{A}_c x^*(t)+\mathbf{B}_{c1}u_1^*+\mathbf{B}_{c2}u_2^*$$

leads to

general form

$$u_i^*=-\mathbf{R}_{ii}^{-1}\mathbf{B}_{ci}^T\mathbf{P}_i(t)$$

$$\frac{d}{dt}\mathbf{P}_i=-\frac{\partial}{\partial x}\mathbf{H}_i\left(x^*,u_1^*,u_2^*,\mathbf{P}_i\right)-\frac{\partial}{\partial u_i}\mathbf{H}_i\left(x^*,u_1^*,u_2^*,\mathbf{P}_i\right).\frac{\partial u_j^*}{\partial x} \quad , \quad (i=1,2)$$

Linear feedback form

$$u_i^*(t)=-\mathbf{R}_{ii}^{-1}\mathbf{B}_{ci}^T\mathbf{K}_i x(t)$$

$$\mathbf{A}_c^T\mathbf{K}_i+\mathbf{K}_i\mathbf{A}_c+\mathbf{Q}_i-\mathbf{K}_i\mathbf{S}_i\mathbf{K}_i-\mathbf{K}_i\mathbf{S}_{\hat{i}}\mathbf{K}_{\hat{i}}-\mathbf{K}_{\hat{i}}\mathbf{S}_{\hat{i}}\mathbf{K}_i+\mathbf{K}_{\hat{i}}\mathbf{S}_{i\hat{i}}\mathbf{K}_{\hat{i}}=\mathbf{0}_n$$

$$where\ \ \mathbf{S}_i=\mathbf{B}_{ci}\mathbf{R}_{ii}^{-1}\mathbf{B}_{ci}^T \quad , \quad \mathbf{S}_{i\hat{i}}=\mathbf{B}_{c\hat{i}}\mathbf{R}_{\hat{i}\hat{i}}^{-1}\mathbf{R}_{i\hat{i}}\mathbf{R}_{\hat{i}\hat{i}}^{-1}\mathbf{B}_{c\hat{i}}^T$$

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simulation

- Vehicle: same sedan vehicle
- Maneuver: lane-change of *4m* at *20 m/s*
- Control Objective: improved handling performance

$$\dot{\theta}_{z,des} = \frac{V_x}{r_{st}\left(L_F + L_B\right)\left(1 + K_{us}V_x^2\right)}\delta_{sw}$$

- Driver LQ structure:

$$Driver: \ \mathbf{Q}_1 = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix}, \ \mathbf{R}_{11} = 1 \ , \ \mathbf{R}_{12} = 0$$

- Vehicle controller LQ structure:

$$DYC: \ \mathbf{Q}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ \mathbf{R}_{21} = 10 \ , \ \mathbf{R}_{22} = 10^{-7}$$

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results

Linear feedback form

$$\boxed{\begin{aligned}&u_i^*(t) = -\mathbf{R}_{ii}^{-1}\mathbf{B}_{ci}^T\mathbf{K}_i x(t) = \mathbf{G}_i x(t)\\ &\mathbf{A}_c^T\mathbf{K}_i + \mathbf{K}_i\mathbf{A}_c + \mathbf{Q}_i - \mathbf{K}_i\mathbf{S}_i\mathbf{K}_i - \mathbf{K}_i\mathbf{S}_{\hat{i}}\mathbf{K}_{\hat{i}} - \mathbf{K}_{\hat{i}}\mathbf{S}_{\hat{i}}\mathbf{K}_i + \mathbf{K}_{\hat{i}}\mathbf{S}_{i\hat{i}}\mathbf{K}_{\hat{i}} = \mathbf{0}_n\\ &\textit{where}\ \ \mathbf{S}_i = \mathbf{B}_{ci}\mathbf{R}_{ii}^{-1}\mathbf{B}_{ci}^T \ \ ,\ \ \mathbf{S}_{i\hat{i}} = \mathbf{B}_{c\hat{i}}\mathbf{R}_{\hat{i}\hat{i}}^{-1}\mathbf{R}_{i\hat{i}}\mathbf{R}_{\hat{i}\hat{i}}^{-1}\mathbf{B}_{c\hat{i}}^T\end{aligned}}$$

- Game Theory:

$$\begin{array}{lccccc} & y & v_y & \theta_z & \dot{\theta}_z & \\ \mathbf{G}_{1(GT)} = [& -0.809 & -0.146 & -8.624 & -0.713 &] \\ \mathbf{G}_{2(GT)} = [& 0 & 504.08 & 0 & -10696 &] \end{array}$$

- Optimal LQ Control:

$$\begin{cases}\mathbf{G}_{1(LQR)} = \begin{bmatrix}-1.0 & -0.215 & -13.63 & -1.286\end{bmatrix}\\ \mathbf{G}_{2(LQR)} = \begin{bmatrix}0 & 129.93 & 0 & -599\end{bmatrix}\end{cases}$$

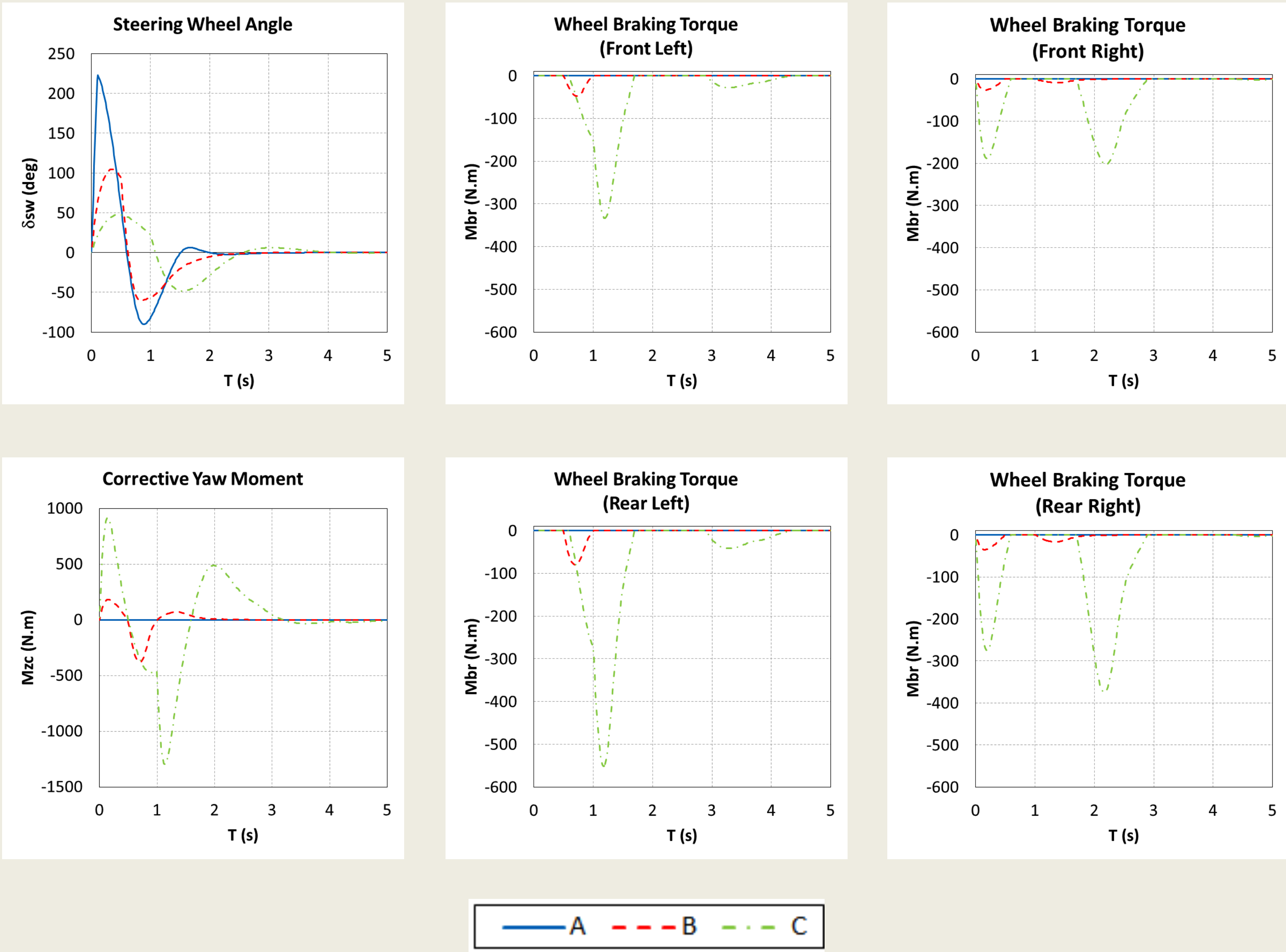
- Scenarios:

(A) driver: LQR & vehicle controller: turned off

(B) driver: LQR & vehicle controller: LQR

(C) driver: GT & vehicle controller: GT

results: continuous-time model



discrete-time interaction model

- Linear vehicle model with road preview

$$\underbrace{\begin{bmatrix} x_d^+ \\ y_r^+ \end{bmatrix}}_{z^+} = \underbrace{\begin{bmatrix} \mathbf{A}_d & 0 \\ 0 & \mathbf{A}_r \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x_d \\ y_r \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} \mathbf{B}_{d1} \\ 0 \end{bmatrix}}_{\mathbf{B}_1} \delta_{sw} + \underbrace{\begin{bmatrix} \mathbf{B}_{d2} \\ 0 \end{bmatrix}}_{\mathbf{B}_2} M_{zc} + \underbrace{\begin{bmatrix} 0 \\ \mathbf{B}_r \end{bmatrix}}_{\mathbf{E}} y_{r(N+1)}$$

- Objective function

$$J_i = \frac{1}{2} \sum_{k=0}^{\infty} \left(z^T \mathbf{Q}_i z + \sum_{j=1}^{2} u_j^T \mathbf{R}_{ij} u_j \right)$$

where

$$\mathbf{Q}_i = \mathbf{N}_i^T \mathbf{Q}_{d\,i} \mathbf{N}_i$$

$$\mathbf{N}_1 = \left[\begin{array}{c|c} \underbrace{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}}_{state\ gains} & \underbrace{\begin{matrix} 0 & -1 & 0 & 0 & \dots & 0 \\ 0 & 1/T_s & -1/T_s & 0 & \dots & 0 \\ 0 & 1/(V_x T_s) & -1/(V_x T_s) & 0 & \dots & 0 \\ -1/(V_x T_s^2) & 2/(V_x T_s^2) & -1/(V_x T_s^2) & 0 & \dots & 0 \end{matrix}}_{road\ preview\ gain} \end{array} \right]_{4\times(4+N)}$$

$$\begin{cases} Y_{ref} = Y_{r1} \\ V_{y,ref} \simeq \dfrac{\Delta Y}{T_s} = \dfrac{Y_{r2} - Y_{r1}}{T_s} \\ \theta_{z,ref} \simeq \dfrac{\Delta Y}{\Delta X} \simeq \dfrac{Y_{r2} - Y_{r1}}{V_x T_s} \\ \dot{\theta}_{z,ref} \simeq \dfrac{\Delta \theta_{z,ref}}{T_s} \simeq \dfrac{\theta_{z,ref}^+ - \theta_{z,ref}^-}{T_s} = \dfrac{Y_{r2} - 2Y_{r1} + Y_{r0}}{V_x T_s^2} \end{cases}$$

$$\mathbf{N}_2 = \left[\begin{array}{c|c} \underbrace{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}}_{state\ gains} & \underbrace{\begin{matrix} 0 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & 0 \end{matrix}}_{road\ preview\ gain} \end{array} \right]_{4\times(4+N)}$$

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discrete LQ Game Theory

- Suppose $\mathbf{P}_i$ satisfy the coupled Riccati equations for discrete linear quadratic games given by

$$\boxed{\mathbf{P}_i = \mathbf{Q}_i - \mathbf{G}_1^T \mathbf{R}_{\hat{i}1} \mathbf{G}_1 - \mathbf{G}_2^T \mathbf{R}_{i2} \mathbf{G}_2 + \left(\mathbf{A} + \mathbf{B}_1 \mathbf{G}_1 + \mathbf{B}_2 \mathbf{G}_2\right)^T \mathbf{P}_i^+ \left(\mathbf{A} + \mathbf{B}_1 \mathbf{G}_1 + \mathbf{B}_2 \mathbf{G}_2\right)}$$

where $i = (1,2)$, and $\hat{i}$ is the counter-coalition, i.e. the player counter-acting to the player with index i, and

$$\boxed{\mathbf{G}_i = -\left(\mathbf{R}_{ii} + \mathbf{B}_i^T \mathbf{P}_i^+ \mathbf{B}_i\right)^{-1} \mathbf{B}_i^T \mathbf{P}_i^+ \left(\mathbf{A} + \mathbf{B}_{\hat{i}} \mathbf{G}_{\hat{i}}\right)}$$

then the following strategy

$$\boxed{u_i^* = -\mathbf{R}_{ii}^{-1} \mathbf{B}_i^T \mathbf{P}_i^+ z^+ \qquad (i = 1,2)}$$

is linear feedback “Nash equilibrium”.

simulation

- Vehicle: same sedan vehicle
- Maneuver: lane-change of *4m* at *20 (m/s)*
- Control Objective: improved handling performance
- Driver/ Vehicle controller LQ structure: same
- Sampling Frequency: 100 Hz
- Integration Frequency: 10,000 Hz

- Scenarios:

  1: no preview time (Tp = *0s*)

  2: short preview time (Tp = *0.3s*)

  3: long preview time (Tp = *1s*)

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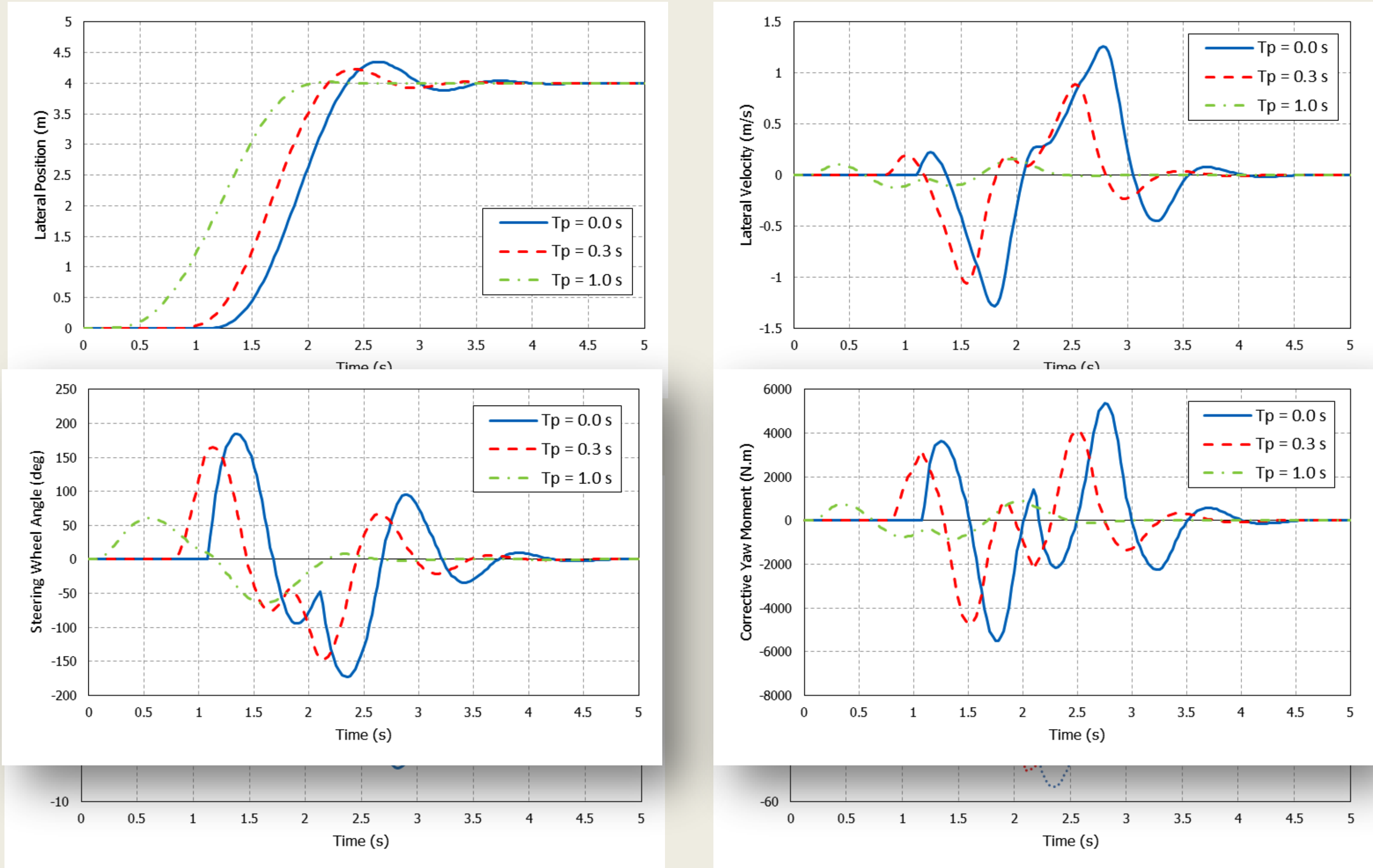
results: discrete-time model



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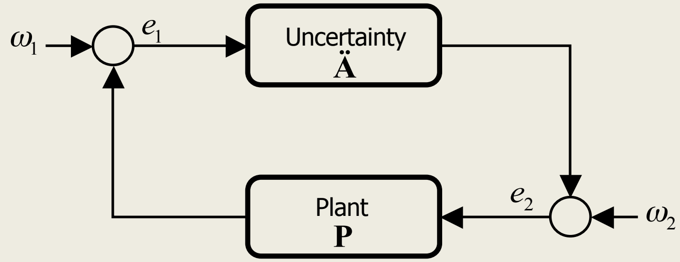
μ−sensitivity analysis

Small Gain Theorem [Zames, 1966]

Consider the feedback interconnection:



Suppose ***P*** *is the plant and let γ>0. Then this feedback interconnection is internally stable for all unstructured uncertainty Δ with $\|\mathbf{\Delta}\|_\infty \le 1/\gamma$ if and only if $\|\mathbf{P}\|_\infty < \gamma$.*

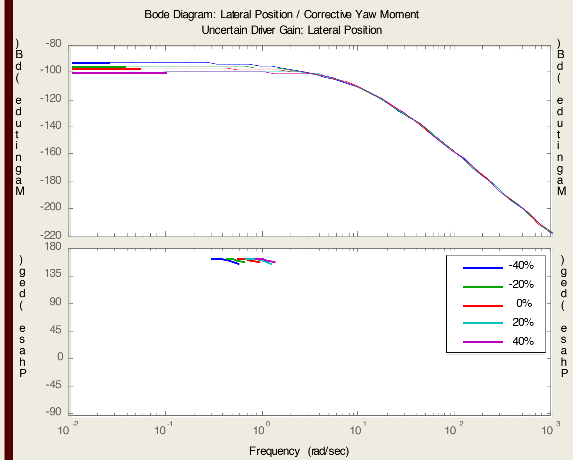
Extension [Packard, 1993]

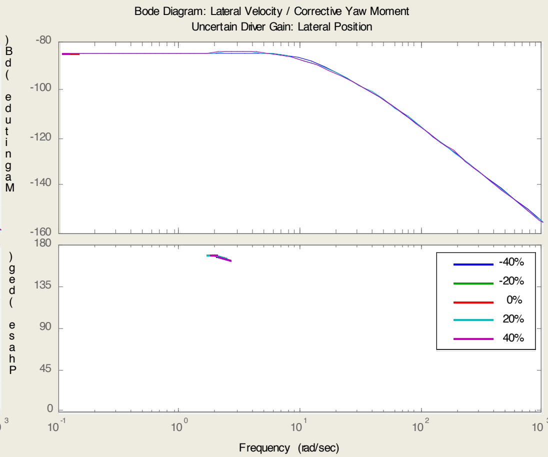
For the same feedback interconnection, assuming that γ>0, this feedback interconnection is internally stable for all structured Δ with $\|\mathbf{\Delta}\|_\infty \le 1/\gamma$ if and only if $\sup_{\delta \in \mathbf{\Delta}} \mu_{\mathbf{\Delta}}(\mathbf{P}) < \gamma$.

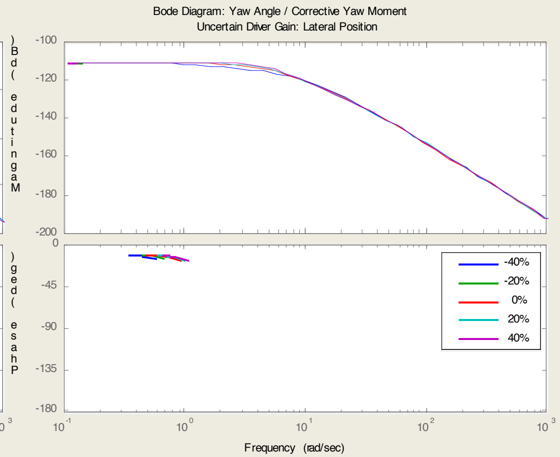
* The structured singular value of **P** with respect to uncertainty structure **Δ** is defined as $\mu_{\mathbf{\Delta}}(\mathbf{P}) = \dfrac{1}{\min\left(\bar{\sigma}(\mathbf{\Delta}) : \left|\mathbf{I} - \mathbf{P}\mathbf{\Delta}\right| = 0\right)}$, $\bar{\sigma}$: maximum singular value of matrix

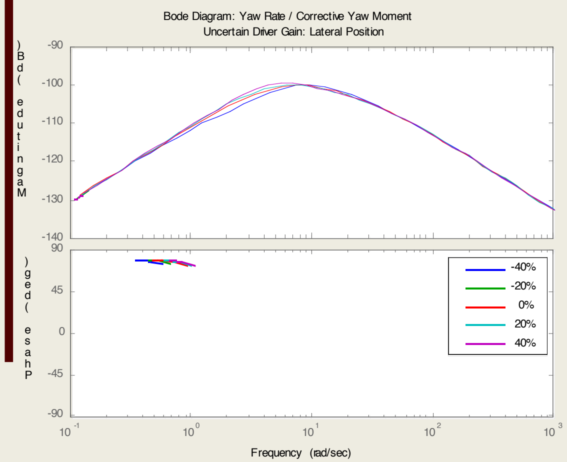
sensitivity analysis of driver gains

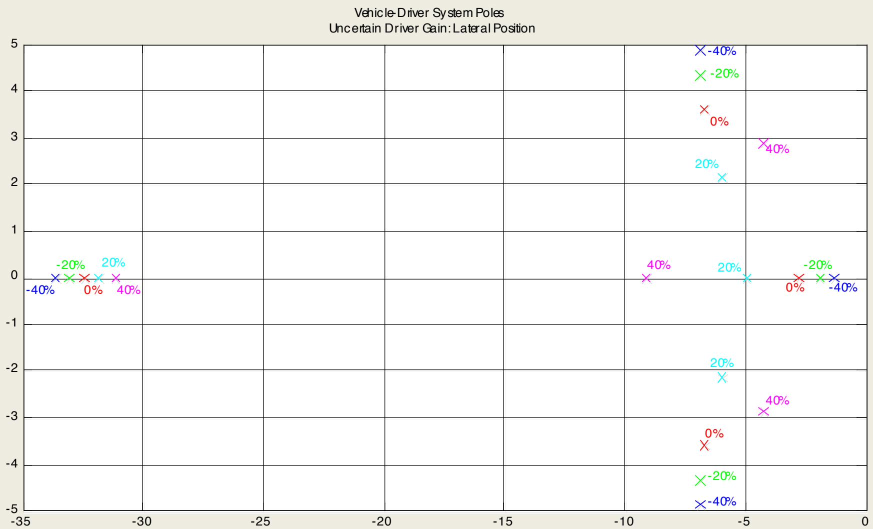
- Effect of uncertainty in <u>lateral position</u> gain





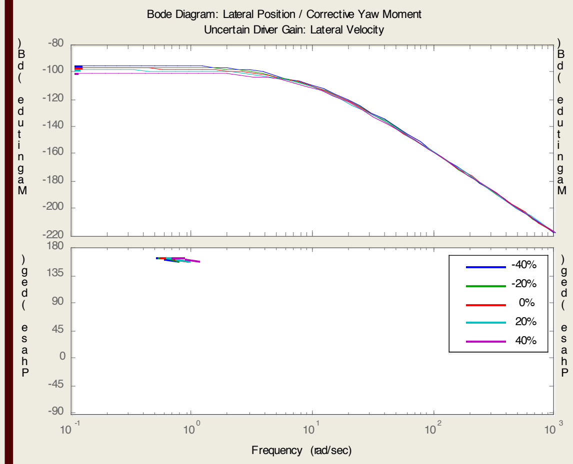


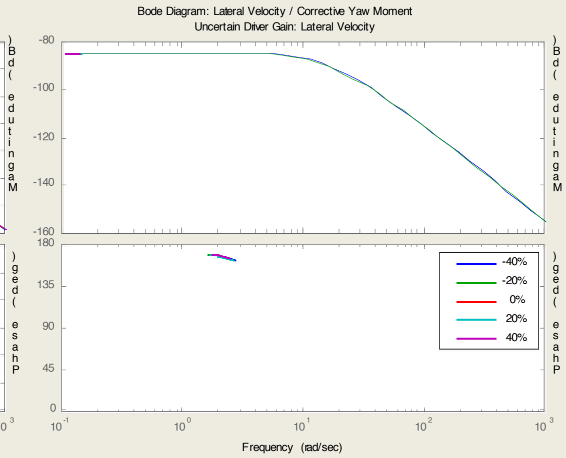


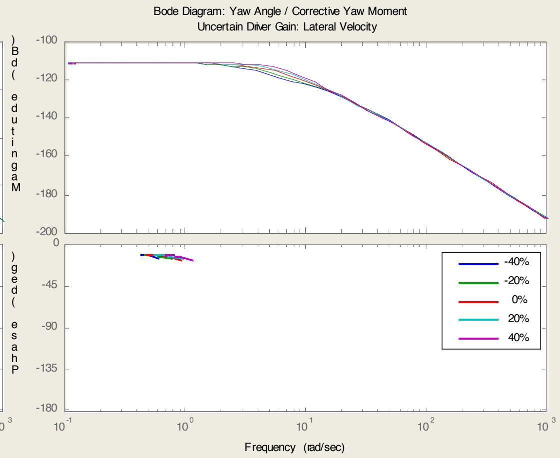


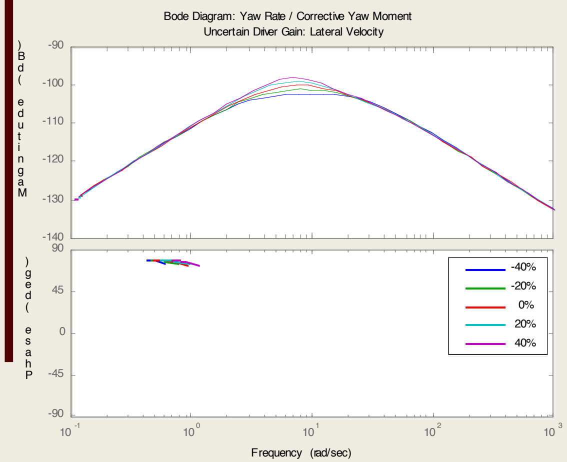
sensitivity analysis of driver gains

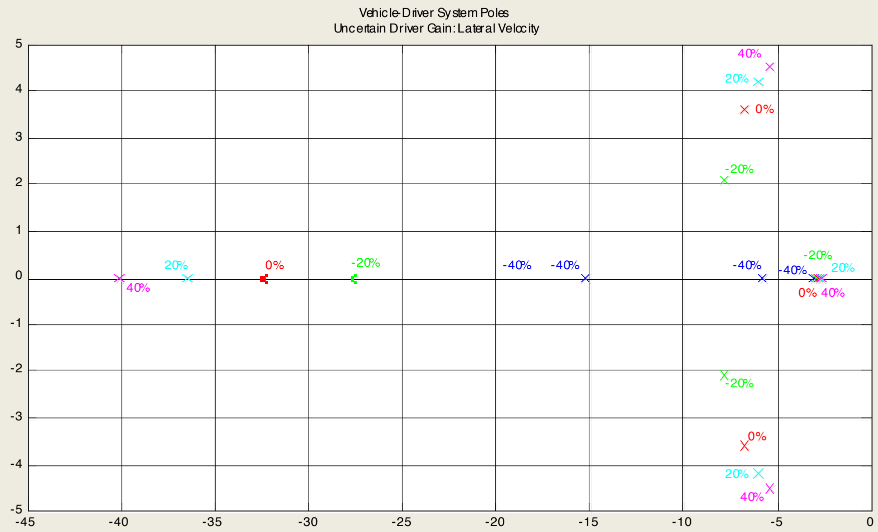
- Effect of uncertainty in lateral velocity gain











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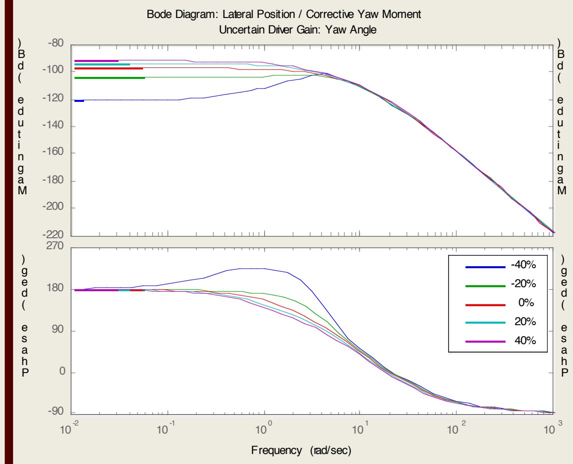
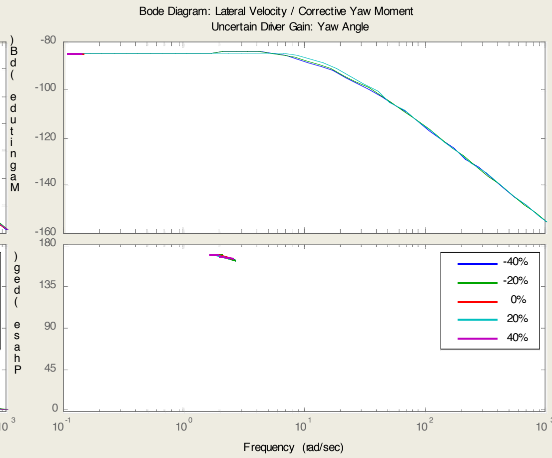
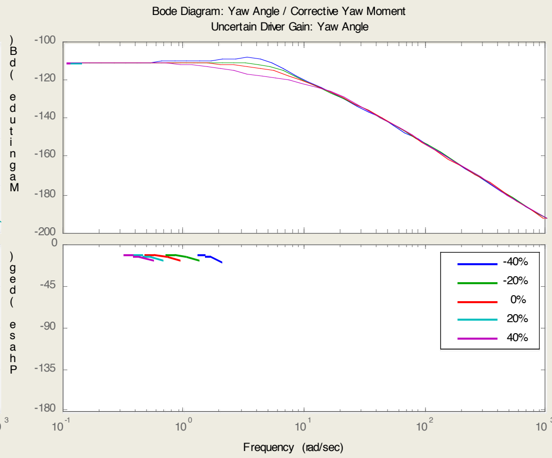
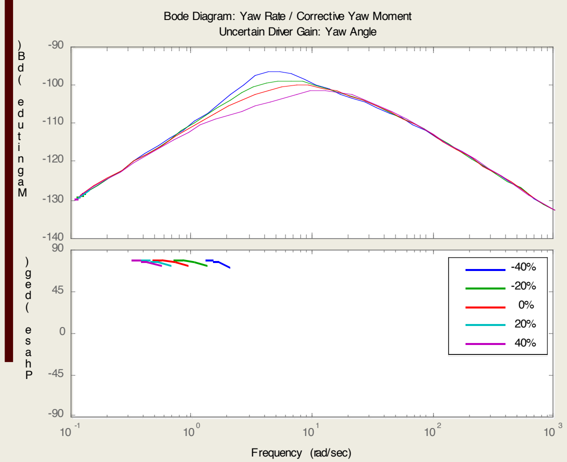
sensitivity analysis of driver gains

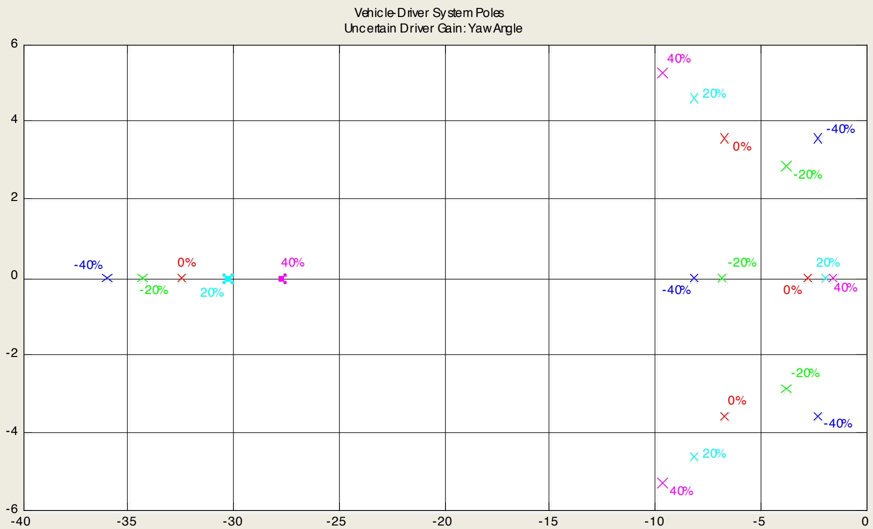
- Effect of uncertainty in yaw angle gain





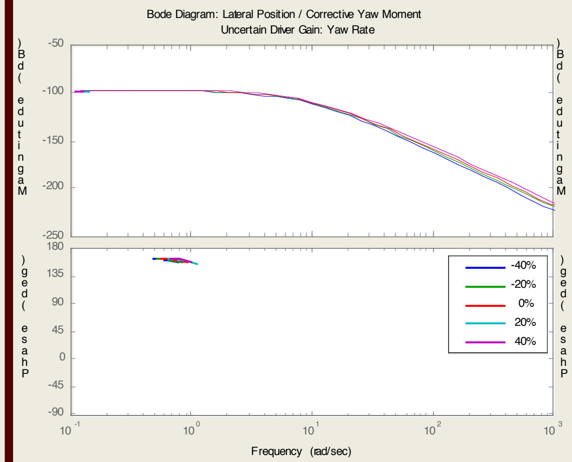


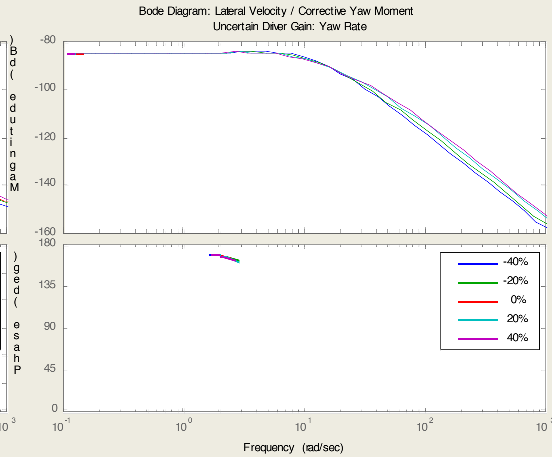


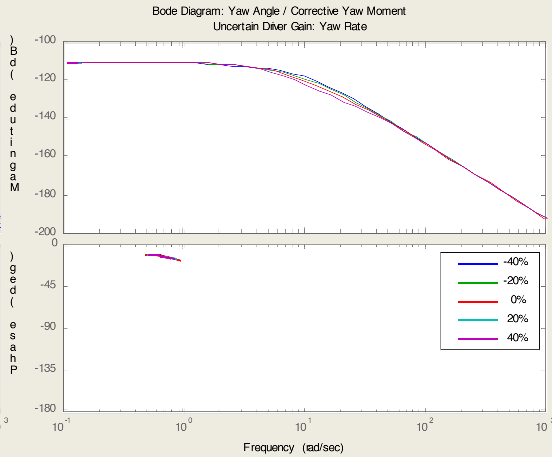


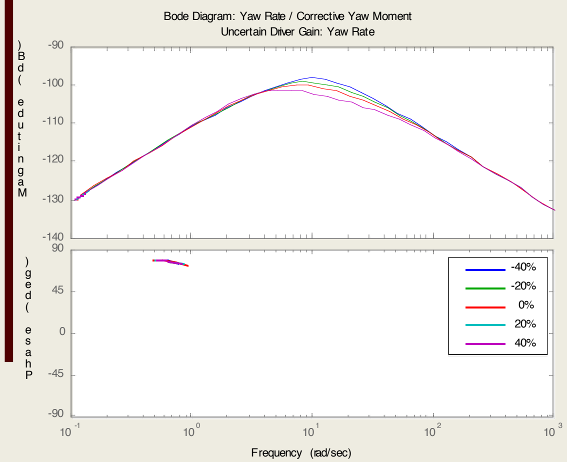
sensitivity analysis of driver gains

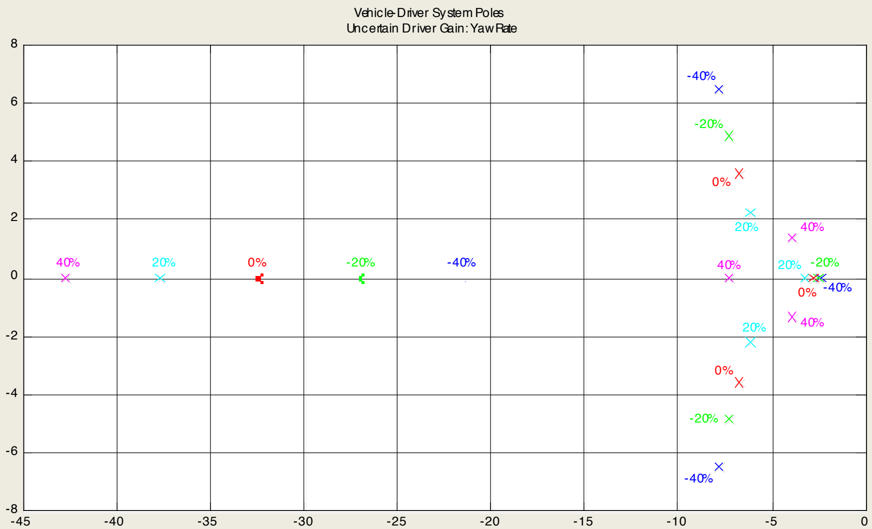
- Effect of uncertainty in yaw rate gain





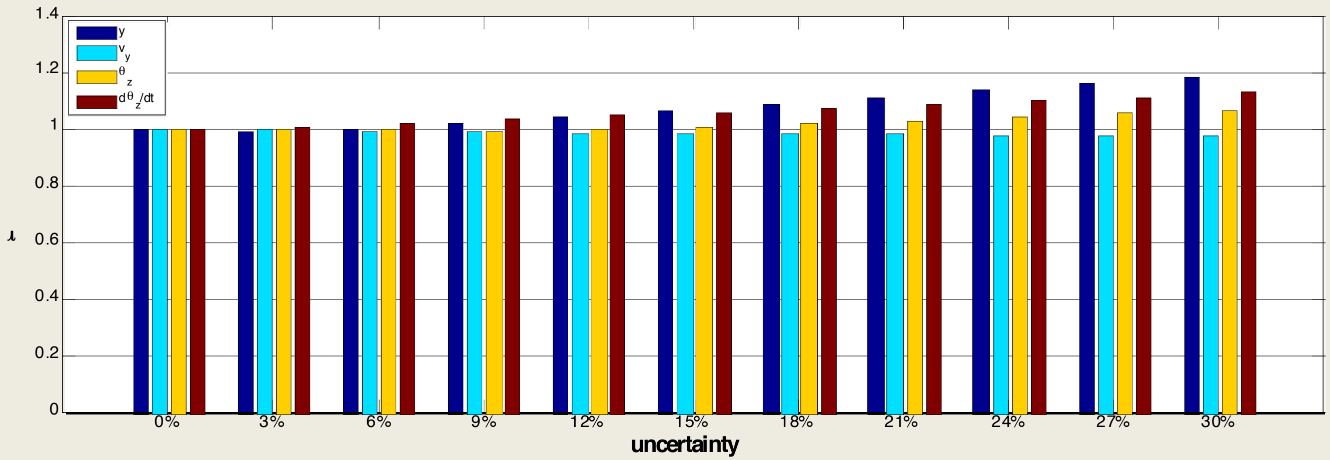




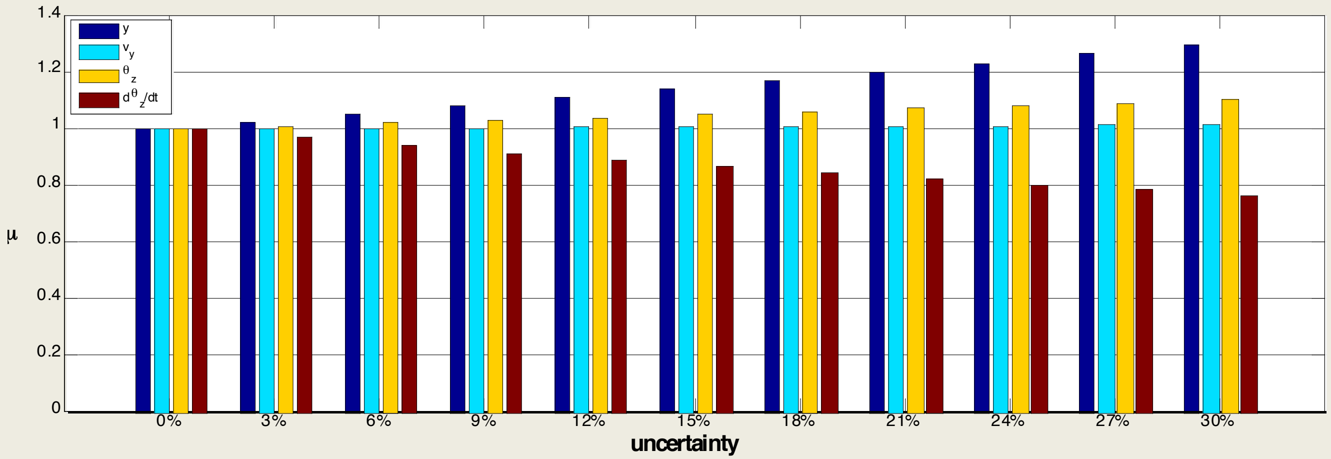


μ−sensitivity plots

Game Theory



LQR



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robust interaction model

- Assume that uncertainties exist in driver model (output)

$$\dot{x}(t) = \mathbf{A}_c x(t) + \mathbf{B}_{c1}\left(\underbrace{\delta_{sw}(t)}_{u_1} + \bar{\delta}(t)\right) + \mathbf{B}_{c2}\underbrace{M_{zc}(t)}_{u_2}$$

- The uncertainty is assume to be smooth disturbances and bounded by

$$\forall t \geq 0, \quad \bar{\delta}(t) \leq \mathbf{F}x(t) + \bar{\delta}_0$$
$$with \;\; \mathbf{F} \in \Re^{1\times 4} \;\; and \;\; \bar{\delta}_0 > 0$$

- To obtain robust control design, the “Integral Sliding Model” approach is applied:

$$\begin{cases} u_1 = u_{o1} \\ u_2 = u_{o2} + u_{\delta} \end{cases}$$

optimal control

disturbance rejecting control

Integral Sliding Mode control design

Uncertain Model :

$$\dot{x}(t) = \mathbf{A}_c x(t) + \mathbf{B}_{c1}\left(\underbrace{\delta_{1sw} + \bar{\delta}(t)}_{u_1}\right) + \mathbf{B}_{c2}\underbrace{M_{zc}(t)}_{u_2}$$

$$\dot{x}(t) = \mathbf{A}_c x(t) + \mathbf{B}_{c1}\left(u_{o1}(t) + \bar{\delta}(t)\right) + \mathbf{B}_{c2}\left(u_{o2}(t) + u_\delta(t)\right)$$

Sliding Surface :

$$s(x,t) = s_o(x,t) + s_\delta(x,t)$$

$$\dot{s}(x,t) = \dot{s}_o(x,t) + \dot{s}_\delta(x,t) = \frac{\partial s_o(x,t)}{\partial x}\dot{x} + \frac{\partial s_o(x,t)}{\partial t} + \dot{s}_\delta(x,t)$$

$$= \frac{\partial s_o(x,t)}{\partial x}\left(\mathbf{A}_c x(t) + \mathbf{B}_{c1}\left(u_{o1}(t) + \bar{\delta}(t)\right) + \mathbf{B}_{c2}\left(u_{o2}(t) + u_\delta(t)\right)\right) + \frac{\partial s_o(x,t)}{\partial t} + \dot{s}_\delta(x,t)$$

Selecting

$$= \mathbf{G}^T\left(\mathbf{B}_{c1}\bar{\delta}(t) + \mathbf{B}_{c2}u_\delta(t)\right)$$

$$s_o(x;t) = \mathbf{G}^T x(t)$$

$$\dot{s}_\delta(x,t) = -\frac{\partial s_o(x,t)}{\partial t} - \frac{\partial s_o(x,t)}{\partial x}\left(\mathbf{A}_c x(t) + \sum_{i=1}^{2}\mathbf{B}_{ci}u_{oi}(t)\right) = -\mathbf{G}^T\left(\mathbf{A}_c x(t) + \sum_{i=1}^{2}\mathbf{B}_{ci}u_{oi}(t)\right)$$

$$s_\delta(x_0,t_0) = -s_o(x_0,t_0)$$

Integral Sliding Mode control design

Lyapunov Candidate :

$$V(x,t) = \frac{1}{2}s^2(x,t) \geq 0 \quad , \quad \forall t$$

$$\dot{V}(x,t) = s(x,t)\dot{s}(x,t) = s(x,t)\left(\mathbf{G}^T\left(\mathbf{B}_{c1}\bar{\delta}(t) + \mathbf{B}_{c2}u_\delta(t)\right)\right)$$

$$= s(x,t)\mathbf{G}^T\mathbf{B}_{c1}\left(\bar{\delta}(t) - \left(\mathbf{F}x(t) + \bar{\delta}_0\right)\mathrm{sgn}(s(x,t))\right)$$

$$\leq s(x,t)\mathbf{G}^T\mathbf{B}_{c1}\left(\bar{\delta}(t) - \mathbf{F}x(t) - \bar{\delta}_0\right)\mathrm{sgn}\left(s(x,t)\right) \leq 0 \quad , \quad \forall t$$

Selecting the disturbance rejecting control as :

$$u_\delta = -\mathbf{B}_{c2}^T\left(\mathbf{B}_{c2}\mathbf{B}_{c2}^T\right)^{-1}\mathbf{B}_{c1}\left(\mathbf{F}x(t) + \bar{\delta}_0\right)\mathrm{sgn}\left(s(x,t)\right)$$

The robust uncertain system :

$$\dot{x}(t) = \underbrace{\left(\mathbf{A}_c - \mathbf{B}_{c1}\mathbf{F}\,\mathrm{sgn}\left(s(x,t)\right)\right)}_{\mathbf{A}_{eq}}x(t) + \mathbf{B}_{c1}u_{o1}(t) + \mathbf{B}_{c2}u_{o2}(t)\underbrace{-\mathbf{B}_{c1}\bar{\delta}_0\,\mathrm{sgn}\left(s(x,t)\right)}_{\omega(t)} + \mathbf{B}_{c1}\bar{\delta}(t)$$

with sliding surface :

$$s(x,t) = \mathbf{G}^T x(t) - \mathbf{G}^T\int_0^t\left(\mathbf{A}_c x(\varepsilon) + \sum_{i=1}^{2}\mathbf{B}_{ci}u_{oi}(\varepsilon)\right)d\varepsilon$$

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Integral Sliding Mode control design

Suppose $\hat{\mathbf{E}}_i$ *satisfy the coupled Riccati equations given by*

$$\mathbf{A}_{eq}^T\mathbf{K}_i + \mathbf{K}_i\mathbf{A}_{eq} + \mathbf{Q}_i - \mathbf{K}_i\mathbf{S}_i\mathbf{K}_i - \mathbf{K}_i\mathbf{S}_{\hat{i}}\mathbf{K}_{\hat{i}} - \mathbf{K}_{\hat{i}}\mathbf{S}_{\hat{i}}\mathbf{K}_i + \mathbf{K}_{\hat{i}}\mathbf{S}_{i\hat{i}}\mathbf{K}_{\hat{i}} = \mathbf{0}$$

and the shifting vectors k_i *satisfy*

$$\mathbf{A}_{eq}^T k_i + \mathbf{K}_i w - \mathbf{K}_i\mathbf{S}_i k_i - \mathbf{K}_i\mathbf{S}_{\hat{i}} k_{\hat{i}} - \mathbf{K}_{\hat{i}}\mathbf{S}_{\hat{i}} k_i + \mathbf{K}_{\hat{i}}\mathbf{S}_{i\hat{i}} k_{\hat{i}} = \mathbf{0}$$

with

$$\mathbf{S}_i = \mathbf{B}_{ci}\mathbf{R}_{ii}^{-1}\mathbf{B}_{ci}^T, \quad \mathbf{S}_{i\hat{i}} = \mathbf{B}_{c\hat{i}}\mathbf{R}_{\hat{i}\hat{i}}^{-1}\mathbf{R}_{i\hat{i}}\mathbf{R}_{\hat{i}\hat{i}}^{-1}\mathbf{B}_{c\hat{i}}^T$$

Then

$$u_{oi}^*(t) = -\mathbf{R}_{ii}^{-1}\mathbf{B}_{ci}^T\left(\mathbf{K}_i x(t) + k_i\right)$$

is a linear feedback Nash equilibrium for the above uncertain game system.

The robust uncertain system:

$$x(t) = \left(\mathbf{A}_c - \mathbf{B}_{c1}\mathbf{F}\,\mathrm{sgn}\left(s(x,t)\right)\right)x(t) + \mathbf{B}_{c1}u_{o1}(t) + \mathbf{B}_{c2}u_{o2}(t) \underbrace{-\mathbf{B}_{c1}\bar{\delta}_0\,\mathrm{sgn}\left(s(x,t)\right) + \mathbf{B}_{c1}\bar{\delta}(t)}_{\omega(t)}$$

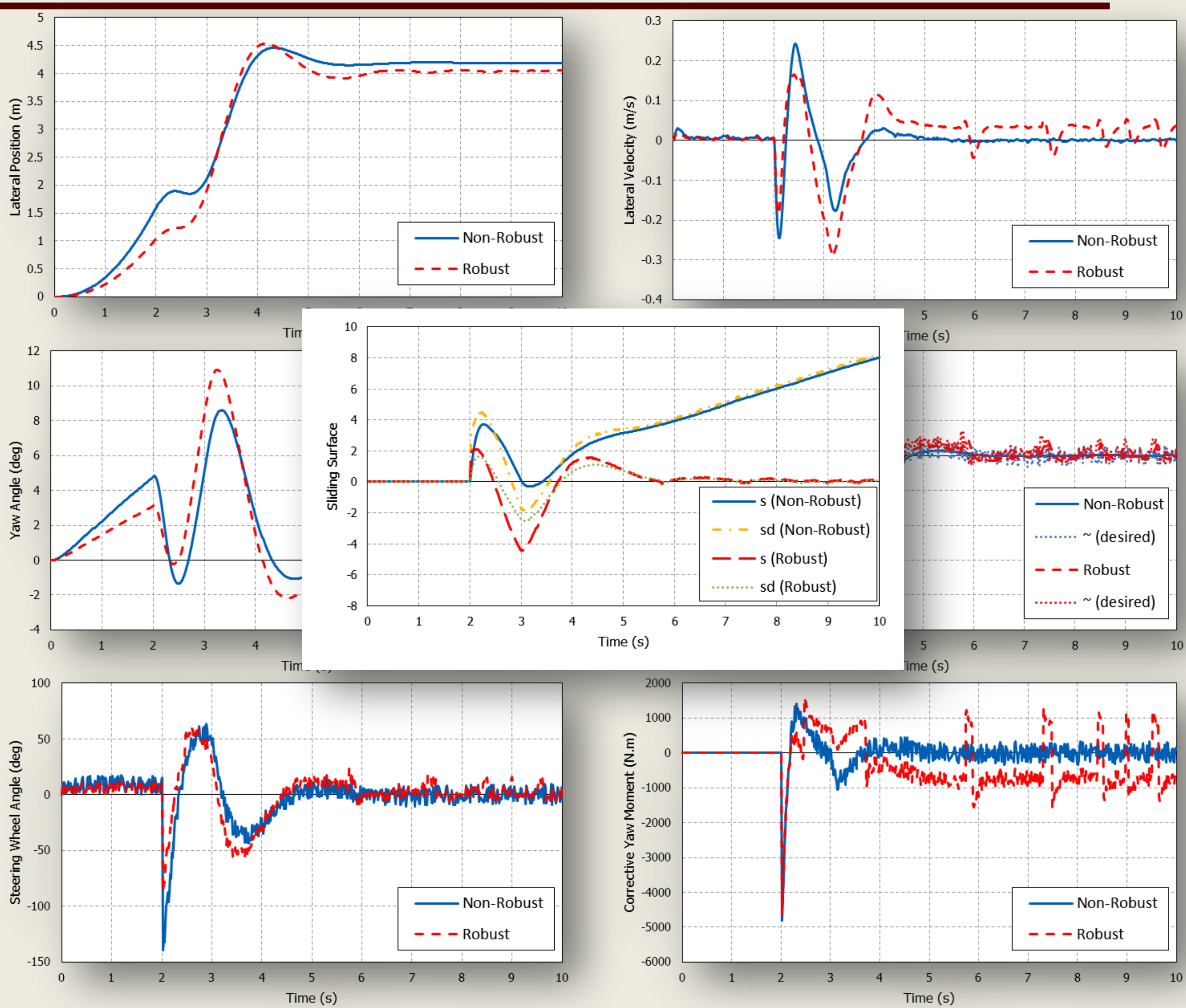
with sliding surface:

$$s(x,t) = \mathbf{G}^T x(t) - \mathbf{G}^T\int_0^t\left(\mathbf{A}_c x(\varepsilon) + \sum_{i=1}^{2}\mathbf{B}_{ci}u_{oi}(\varepsilon)\right)d\varepsilon$$

simulation

- Vehicle: same sedan vehicle
- Maneuver: lane-change of *5m* at *20 m/s*
- Control Objective: improved handling performance
- Driver/ Vehicle controller LQ structure: same
- Driver steering angle uncertainty: $\mathbf{F} = (20\%)\mathbf{B}_1 \;,\; \bar{\delta}_0 = 10°$

results



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conclusions

- Extension of discrete preview driver steering control model to include lateral velocity and yaw rate was developed
- Introduction of modified Cruise Control and Differential Braking systems for application of the proposed control algorithm
- A new structure for optimal linear vehicle steering and yaw control has been devised based on linear quadratic Game Theory
  - * Continuous-time interaction model
  - * Discrete-time interaction model with preview-time
  - * Robust interaction model with sensitivity analysis

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