OPTIMISING ESTIMATORS
FOR
OPERATIONAL MODAL ANALYSIS

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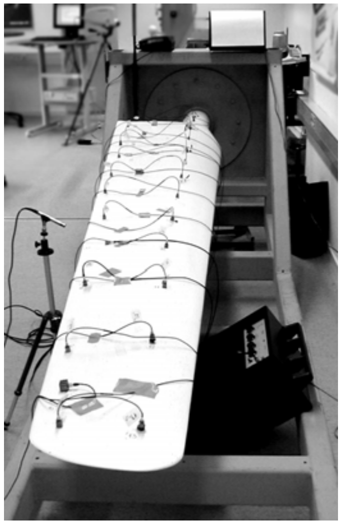
November 10, 2005



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Contents

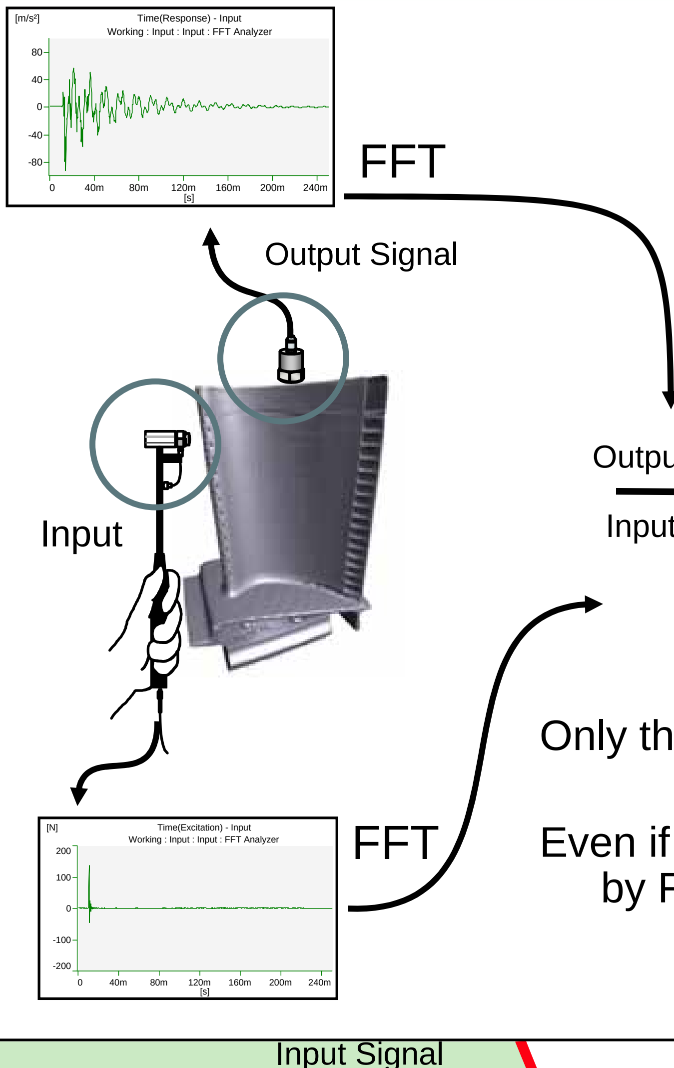
Contents

Operational Modal Analysis – *(OMA)*

- Determination of Modal Model by **response testing only**
  - No measurement of input forces required
  - Measurement procedure similar to
    Operational Deflection Shapes but modal parameters are achieved
- Determination of Modal Model under **operational conditions**
- Typical Applications (OMA)
  - **Cars** - on road testing
  - **Aircraft -** during flight test
  - **Wind Turbines** – in operation
  - **Automotive sub-components** – mounted in the real environment
  - **Buildings** – under wind-load
  - **Electronic instrumentation** – drop test
  - …………

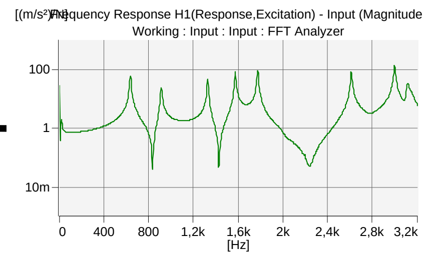
Mobility Measurements (*Traditional Modal Testing*)

- **Input** Force is measured
- **Output** Response is measured
- Output is related to Input by FRF estimators
- FRF is independent of the input force



Input/Output modal analyses

Output Spectrum
―――――――
Input Spectrum



Structural System

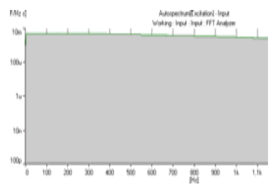
Only the Modes of the System need to be identified

Even if the system is producing noise this is handled by FRF estimators as H_1, H_2 , H_3

Operational Modal – *(The Combined Model)*

If the system is excited by white noise
the output spectrum contains full information of the structure
as all modes are excited equally

Output only modal analysis

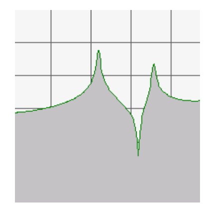


Force Spectrum

→

Structural System

→

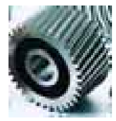


Output Spectrum

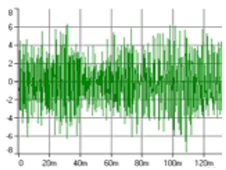
But this is in general ***not*** the case!

Operational Modal – *(The Combined Model)*

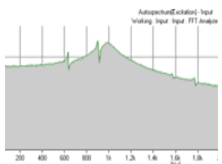
Output only modal analysis



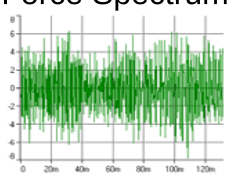
Rotating Parts create Harmonic vibrations



Computational Noise

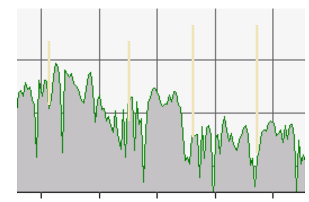


Force Spectrum



Measurement Noise

Structural System



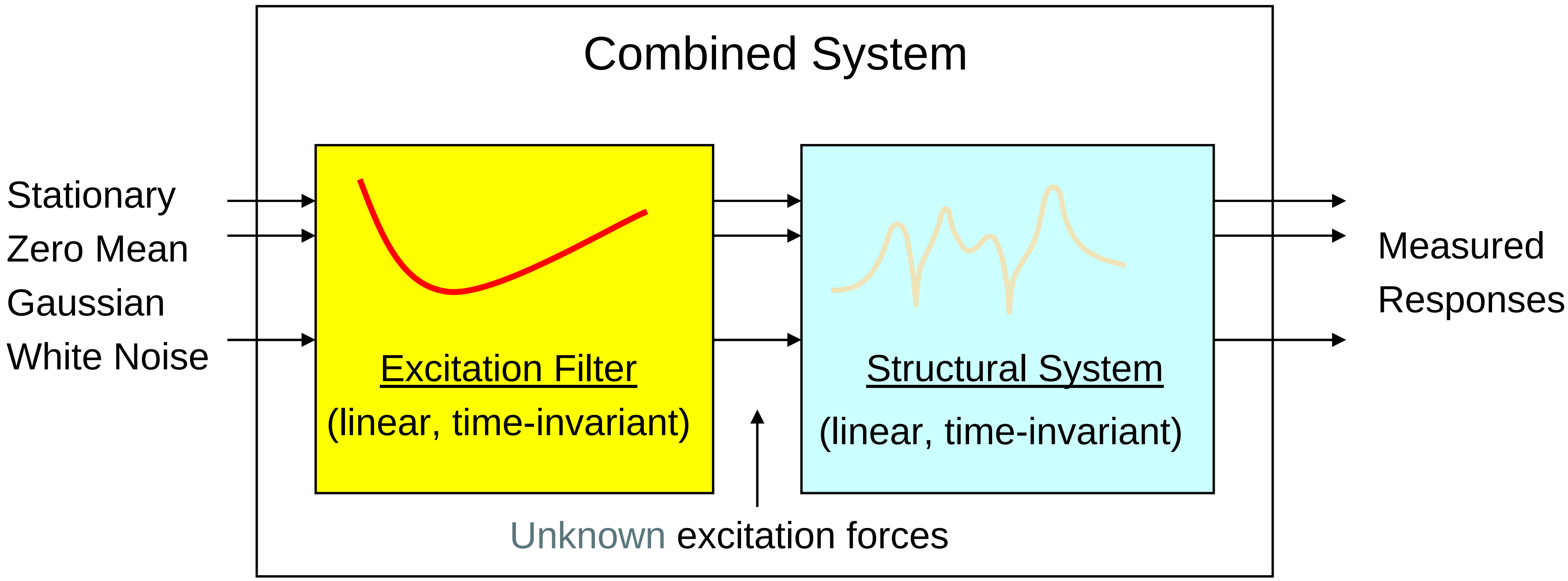
Combined Spectrum

The "Modes" in the combined spectrum contains information of

- The system under test (Physical Modes)
- Input Force (Non-physical Modes)
- Noise (Non-physical Modes)
- Harmonics (Non-physical Modes)

Combined System Model *(analysis procedure)*

Model of the **Combined System** is estimated from measured responses



Modal Model of Structural System
extracted from estimated model of Combined System

Assumptions

Mathematical

- Stationary input force signals can be approximated by filtered zero mean Gaussian white noise
  - Signals are completely described by their correlation functions or auto- & cross-spectra
  - Synthesized spectral densities and correlation functions are similar to those obtained from experimental data

Practical

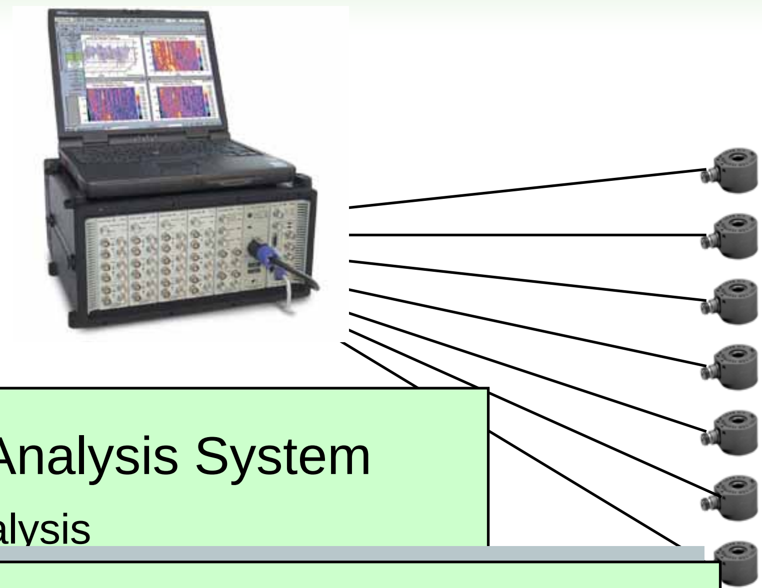
- Broadband excitation
- All modes must be excited

- **Un-scaled** (Non Calibrated) Modal Model,

although method are proposed

Contents

Instrumentation



PULSE™ Multi Analysis System

FFT for pre-analysis

PULSE™ Modal Test Consultant

Organizing the Measurements

DOF-Information

Raw Time Data

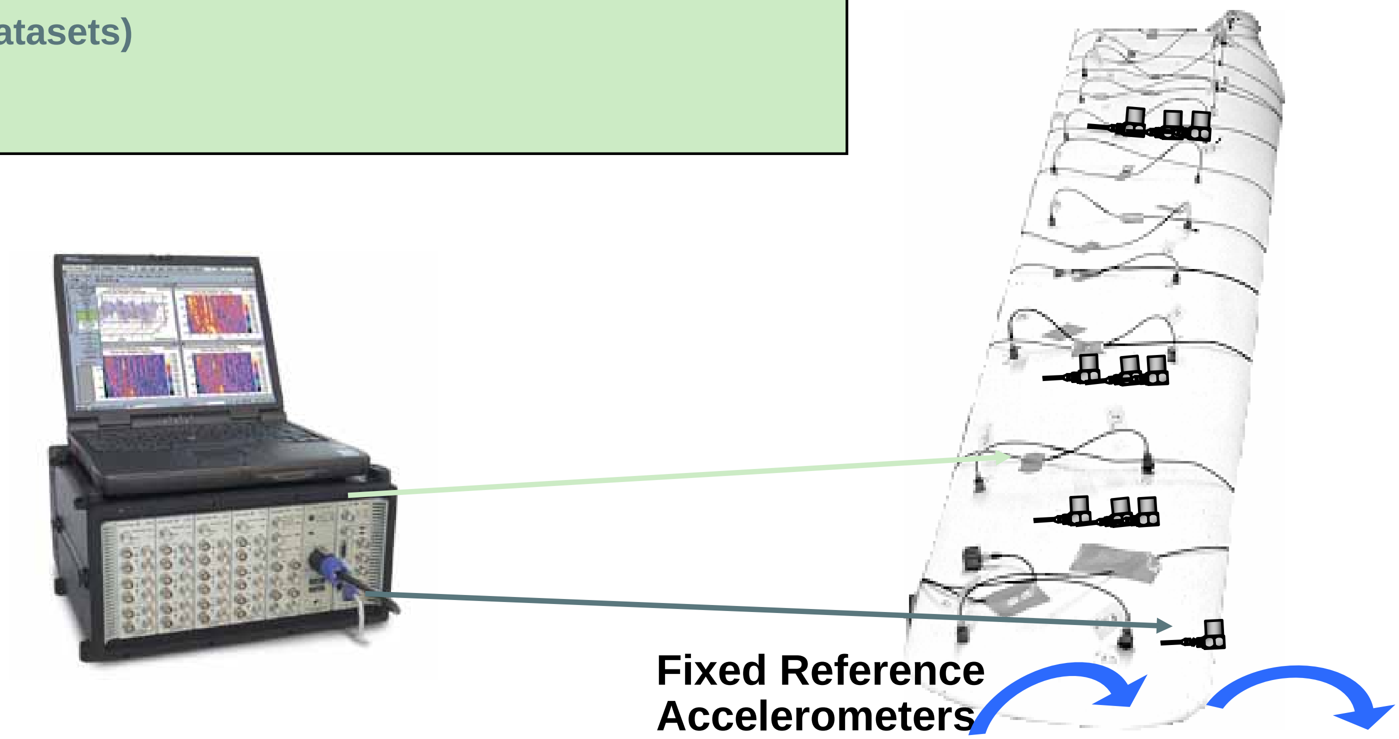
Operational Modal Analysis

Modal Parameter Estimation

Measurements

The Reference is a tri-axial Accelerometer mounted at a Fixed Position

A Group of Accelerometers is mounted at different positions (datasets)



Contents

Frequency Domain Decomposition (FDD)

Determination of complete Modal Model from Responses only

FDD procedure:

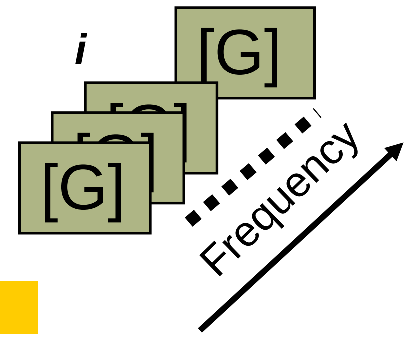
- Power Spectral Density (PSD) estimation
- Singular Value Decomposition (SVD) of PSD matrix
- Identification of Single Degree of Freedom (SDOF) models from SVD
- Modal Parameter identification from SDOF models

Frequency Domain Decomposition (FDD)

Singular Value Decomposition of PSD

$$G_{yy}(j\omega_i) = \sum_k \frac{d_k}{j\omega_i - \lambda_k}\phi_k\phi_k^T + \frac{\bar{d}_k}{j\omega_i - \bar{\lambda}_k}\bar{\phi}_k\bar{\phi}_k^T = \sum_k s_k\phi_k\phi_k^T + s_k\bar{\phi}_k\bar{\phi}_k^T$$

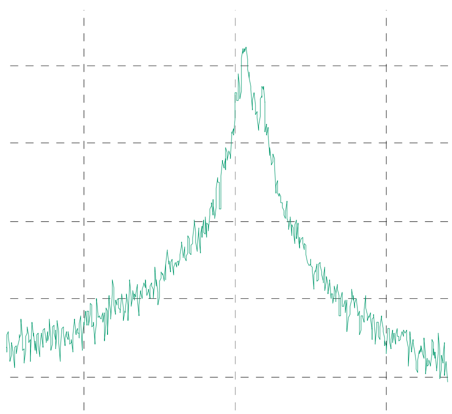
- s_k is constant and real for a given frequency
- SVD performed for each frequency



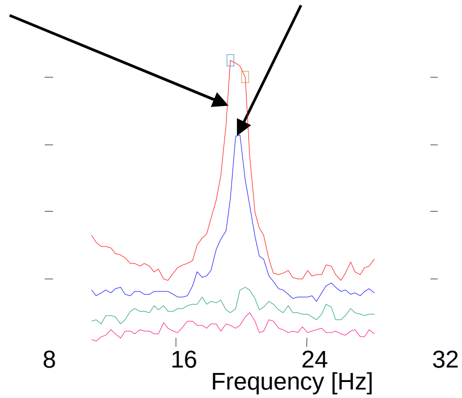
Using SVD to extract modal parameters from PSD

S_1: At least one mode exists

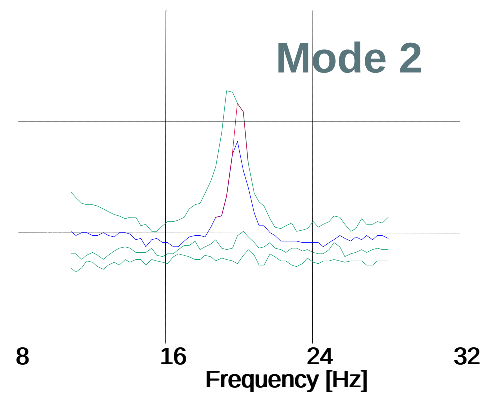
S_2: At least two modes exist



PSD Mag.



SVD of PSD



Decoupled Modes

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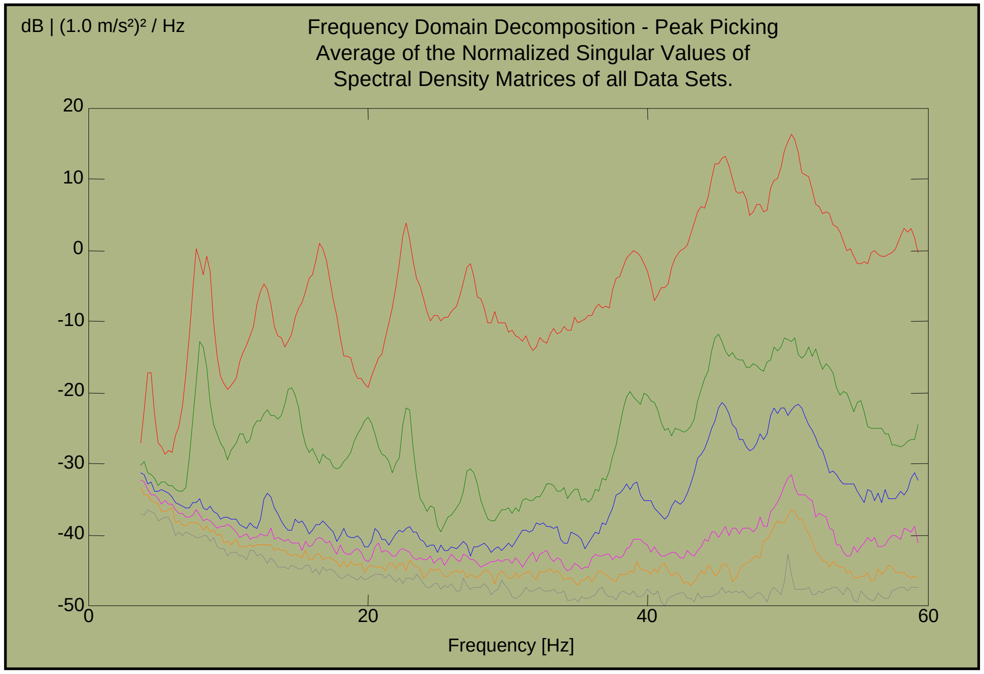
FDD Method

Singular Values plotted as function of frequency

Only three singular values carry information of the modes



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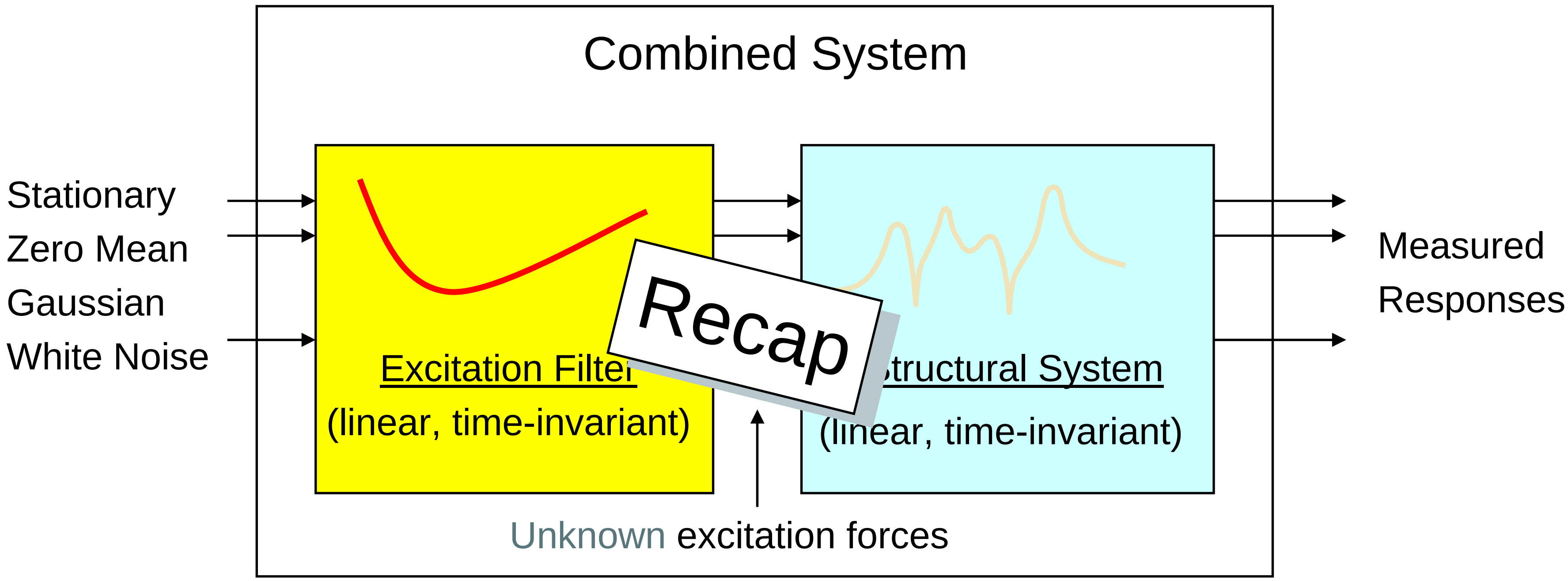
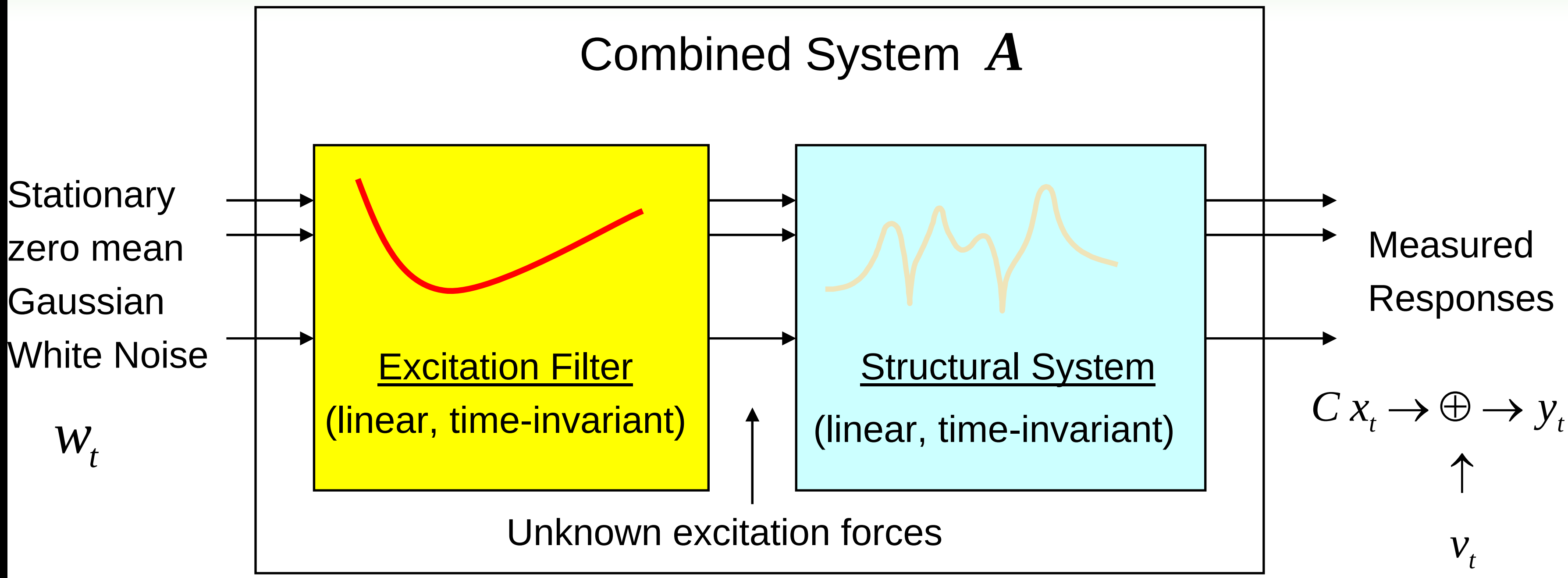
Combined System Model *(analysis procedure)*

Model of the **Combined System** is estimated from measured responses



Modal Model of Structural System
extracted from estimated model of Combined System

Combined System Model used in SSI



Discrete-time Stochastic State Space Model

State Equation	$x_{t+1} = Ax_t + w_t$	Model of the dynamics of the system
Observation (Output) equation	$y_t = Cx_t + v_t$	Model of the output of the system

$\boldsymbol{w_t}$: Process noise - $\boldsymbol{v_t}$: Measurement noise - Model order: Dimension of ***A***

Stochastic Subspace Identification (SSI)

Modal parameter extraction from SSI

Discrete-time Stochastic State Space Model

$$x_{t+1} = Ax_t + w_t$$
$$y_t = Cx_t + v_t$$

$\boldsymbol{w_t}$: Process noise
$\boldsymbol{v_t}$: Measurement noise

Innovation form

$$\hat{x}_{t+1} = A\hat{x}_t + Ke_t$$
$$y_t = C\hat{x}_t + e_t$$

$\mathbf{e_t}$: Innovation (white noise)
$\boldsymbol{K}$: Kalman gain (noise model)

Modal decomposition

$$z_{t+1} = [\mu_i]z_t + \Psi e_t$$
$$y_t = \Phi z_t + e_t$$

μ_i Eigenvalues, Modal frequency and damping

Φ Mode shape

Ψ Right hand side vector, Modal distribution of $\boldsymbol{e}$

Stochastic Subspace Identification (SSI)

Estimation of state vectors:

$$\hat{X}_i = [\hat{x}_i \; \hat{x}_{i+1} \; \cdots \; \hat{x}_{i+j-1}]$$

Projection Channels

S_1

$$\text{SVD:} \quad W_1 O W_2 = [U_1 U_2] \begin{bmatrix} s_1 & 0 & 0 & 0 & . & . & . & 0 \\ 0 & s_2 & 0 & . & . & & & . \\ 0 & 0 & . & . & & & & . \\ 0 & . & . & s_\rho & & & & . \\ . & . & & & 0 & & & . \\ . & & & & & 0 & & . \\ . & & & & & & 0 & . \\ 0 & . & . & . & . & . & . & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 S_1 V_1^T$$

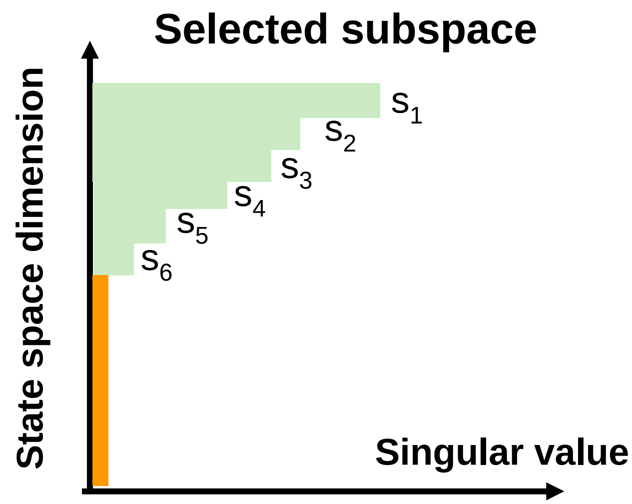
O: Compressed input format matrix

W_1, W_2: Weighting matrices

S_1 : Subspace matrix

$\hat{X}_i$ is calculated from

$$U_1 S_1 V_1^T = U_1 S^{1/2}{}_1 \cdot S^{1/2}{}_1 V_1^T = W_1 \Gamma_i \cdot \hat{X}_i W_2$$



Projection Channels Procedure

- Decide number of projection channels P
- Calculation of the correlation coefficients between the different measurement channels

$$C_{ij} = \frac{E\left[y_i(t)y_j(t)\right]}{\sqrt{E\left[y_i(t)y_i(t)\right]}\sqrt{E\left[y_j(t)y_j(t)\right]}} \quad ,i=1..p$$

- Find the channel i that correlates **most** with **all** the other channels

$$W_i = \sum_{j=1,\, j\neq i}^{p} \left|C_{ij}\right| \quad ,i=1..p$$

- Find the rest (P-1) of the projection channels that correlates **least** with previous found projection channel

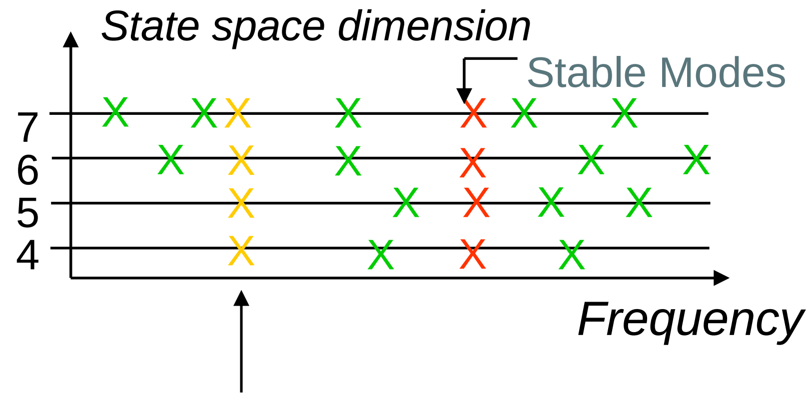
Stochastic Subspace Identification (SSI)

- Parametrical Modal estimation requiring apriory knowledge of Model Order
- Physical Modes as well as Non-physical Modes are estimated

How can we separate Physical Modes from Non-physical Modes?

Physical modes are repeated for multiple Model orders!

Stabilization Diagram



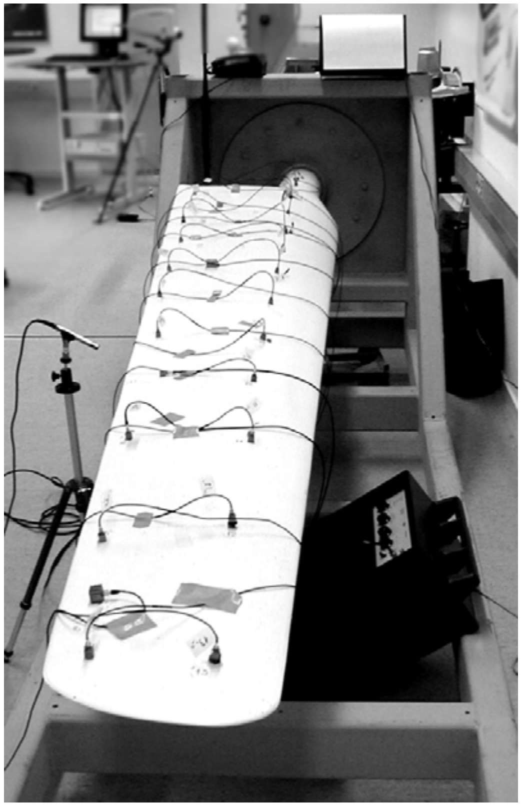
X Estimated parameters not fulfilling apriori knowledge of damping

X Stable modes are repeated in two consecutive models fulfilling user defined criteria

X Remaining modes are considered as unstable

Contents

Test Set-up



- A 1:5 scale wind turbine wing
- 24 accelerometers are mounted in two rows along the wing
- Two time recordings taken,
  - accelerometers perpendicular to the surface
  - pointing in the direction of rotation
  - Tri-axial Accelerometer as reference
- Acoustic load by means of a loudspeaker
- Nyquist Frequency: 256 Hz
- Duration of Data: 60 seconds

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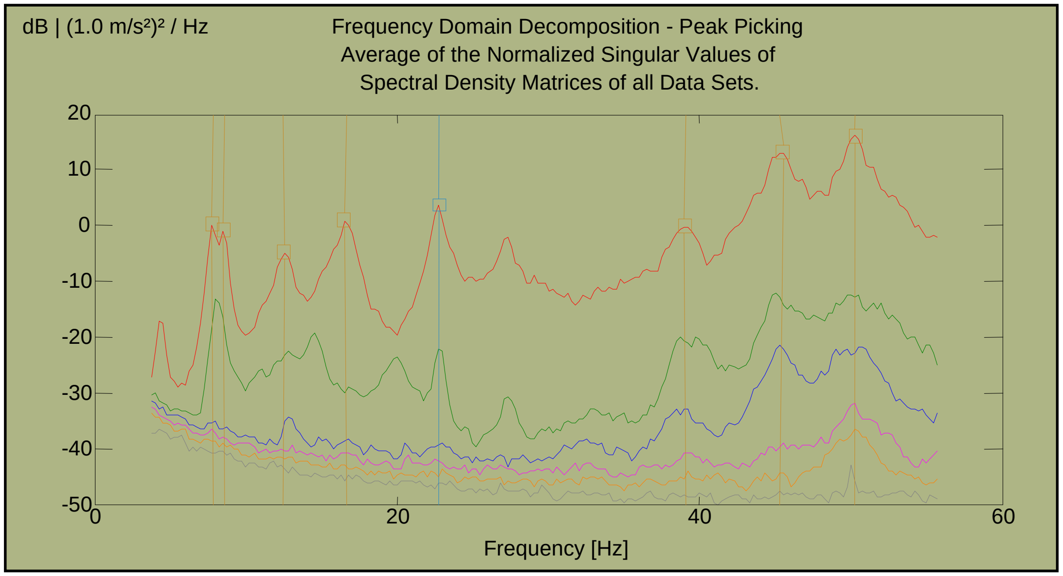
Enhanced Frequency Domain Decomposition

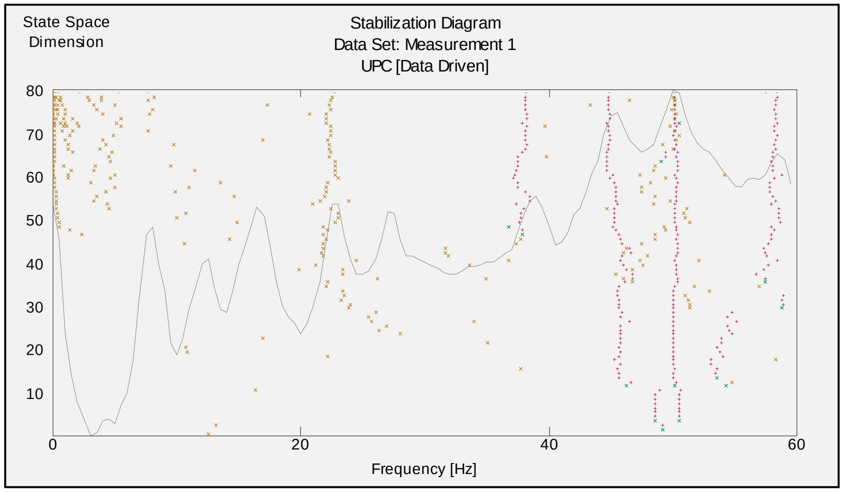
Peak-Picking on the average of the normalized singular values of the PSD Matrix for all datasets



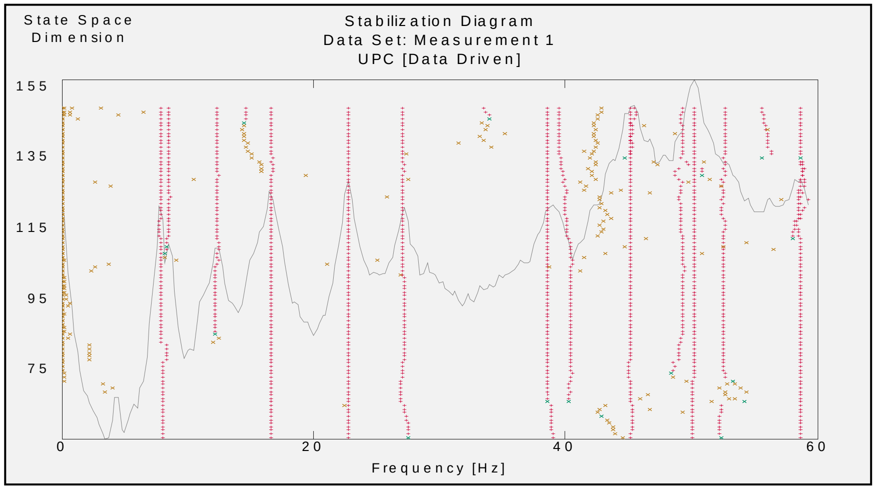
Results

Stabilisation Diagrams

- 0-60 Hz
- 0-40 Modes

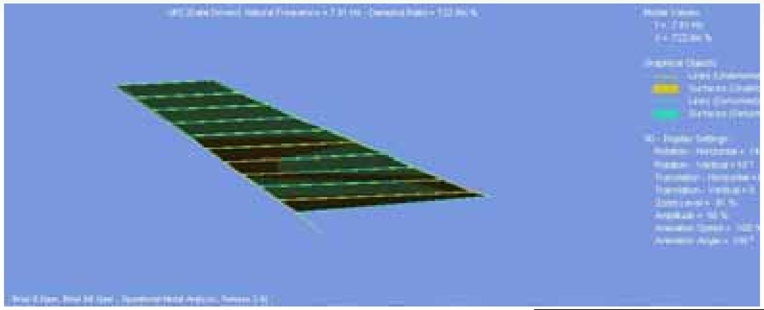


Stabilisation diagram without channel projection.

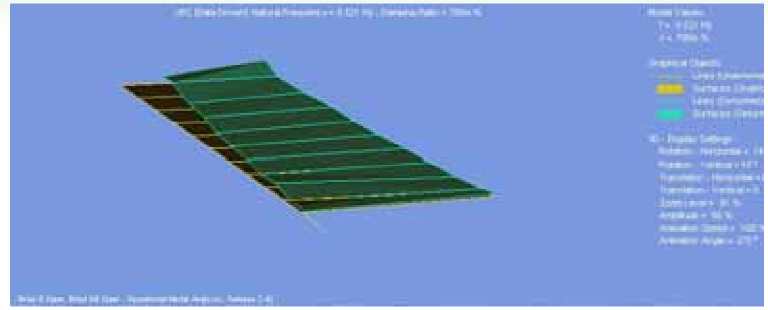


Stabilisation diagram with channel projection=3

Mode Shapes



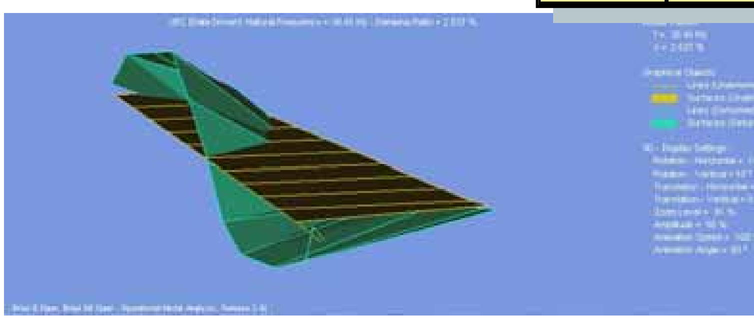
Mode 1



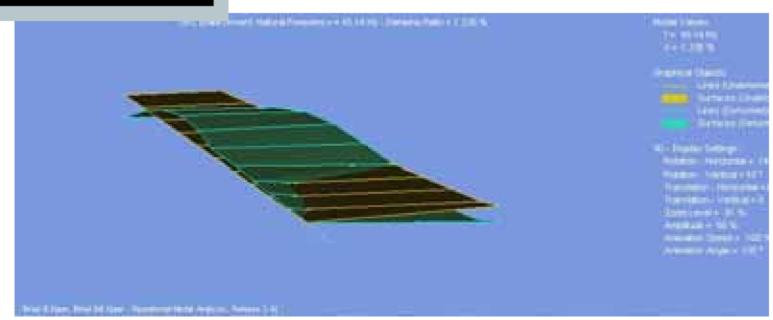
Mode 2

Mode	Frequency [Hz]	Damping Ratio [%]
Mode 1	7.81	722.8m
Mode 2	8.52	786m
Mode 3	12.54	2.765
Mode 4	16.58	2.515
Mode 5	22.75	1.239
Mode 6	26.88	2.286
Mode 7	38.45	2.537
Mode 8	39.39	2.322
Mode 9	45.14	1.336
Mode 10	49.29	2.55
Mode 11	50.45	817.8m

Mode 7



Mode 9



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Conclusion

Operational Modal Analysis has many advantages over classical Modal Analysis

- No time-consuming set-up of shakers.
- Measurements performed in a realistic environment
- Both **Local** and **Global** Modes can be clearly identified as a result of the same test.

Implementing Channel Projection Improves the Technique further by:

- Minimizing the redundant information
- Clearer identification of the Modes
- Lowering the required amount of data
- Decreasing calculation time